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GOVERNMENTS' DEBTS AND PUBLIC GOODS IN A MULTI-COUNTRY GROWTH MODEL WITH TRADABLE AND NON-TRADABLE GOODS

Prof. Wei-Bin Zhang

Ritsumeikan Asia Pacific University, Japan

Abstract: *This study deals with dynamic relationships between global growth, trade, economic structural change, and government's debts. Government debts are seldom theoretically modelled in the literature of global economic growth theory. We introduce governments' debts and endogenous public good supplies into a general dynamic equilibrium growth model with multiple countries and free trades between countries. The model is developed by integrating the Solow-Uzawa growth model, the Oniki–Uzawa trade model, and Diamond's growth model with government's debt within a comprehensive framework. The model synthesizes these well-known economic models with Zhang's utility function to determine household behavior. It is built for any number of national economies. Each national economy consists of one tradable, one non-tradable and one public sector. The model describes a dynamic interdependence between wealth accumulation, and division of labor, governments' debts, national debts, and wealth and capital distribution under perfect competition. We demonstrate that the dynamics of the J -country world economy can be described by $2J$ differential equations. We simulate the model, demonstrating the existence of an equilibrium point, and showing instability of the equilibrium point. We also demonstrate how changes in some parameters affect short-run global economic development and the equilibrium point. Our comparative dynamic analyses provided some important insights into interactions between global economic growth, resource distributions, economic structures, and governments' debts.*

Keywords: trade pattern, government's debt, tradable and non-tradable, economic growth, wealth accumulation

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Contact address:
Prof. Wei-Bin Zhang
Ritsumeikan Asia Pacific University
1-1 Jumonjibaru, Beppu-Shi, Oita-ken、874-8577 Japan
(Home Tel: 0977-73-9787; Office Tel: 0977-78-1020; Fax: 0977-78-1123;
E-mail: wbz1@apu.ac.jp)

1. INTRODUCTION

Government debts have become an important issue in developing as well as developed economies. To comprehend national debts are not easy as they have complicated relations with national output, taxes, taxation structures, government's social and economic activities, population structure, international trade, and economic growth. Although some theoretical models have been proposed to study relations between national debts and growth, these models mostly deal with very simple economic structure with simplified taxation. This study deals with issues related to debts and economic growth with international trade in the neoclassical growth framework. We deal with dynamics of debts by considering government expenditure and different taxes in a competitive global economy with any number of national economies. We introduce public goods to the growth model. There are a few models which treat productive fiscal policy as a determinant of persistent economic growth (Barro, 1990; Turnovsky, 2000, 2004; Gómez, 2008; and Park, 2009). In our approach only the governments are responsible for the provision of public goods. Each country's government financially supports the public sector to supply public good. The governments use different taxes and public debt to finance the public sectors. The government has a set of control measures including the total expenditures and tax rates on the industrial sector's output, the service sector's output, the wage income, the consumption and the interest income. Almost of all the recent theoretical literature of dynamic interactions between economic growth and public debts use either the Ramsey framework in continuous time (Cohen and Sachs, 1986; Blanchard and Fischer, 1989; Barro *et al*. 1995; Semmler and Sieveking, 2000; Guo and Harrison, 2004; and Giannitsarou, 2007) or the OLG modeling framework in discrete time (Diamond, 1965; Farmer, 1986; Turnovsky and Sen, 1991; Azariadis, 1993; de la Croix and Michel, 2002; and Chalk, 2000). Different from these traditional approaches, this study applies Zhang's approach to household decision.

This study is primarily concerned with dynamics of governments' debts in an analytical framework with interactions among wealth and physical capital accumulation, trade patterns, governments' debts, and national debts between multiple national economies with endogenous public good policy and different tax rates. In our approach global economic growth is mainly enforced by wealth and physical capital accumulation. The global growth mechanism of physical accumulation is based on the Solow growth model. We describe international trade patterns on the basis of the dynamic trade models with accumulating capital developed by Oniki and Uzawa and others (for instance, Oniki and Uzawa, 1965; Frenkel and Razin, 1987; Sorger, 2002; and Nishimura and Shimomura, 2002). The Oniki-Uzawa model is constructed for the two-country with two goods. We use tradable good and non-tradable rather than capital goods and consumer goods as in the Oniki-Uzawa model. Distinction between tradable good and non-tradable good is significant for explaining the terms of trade (Mendoza, 1995; Stockman and Tesar, 1995). This paper introduces governments' debts into the multi-country growth models with international trade and economic structure with tradable good and non-tradable good. It synthesizes two papers recently proposed by Zhang (2014, 2016). Zhang (2014) proposes a global growth model with tradable and non-tradable goods. Zhang deals with a national economy with government debt. Different from the growth models with the Ramsey approach, we use the alternative utility function proposed by Zhang (1993) to determine saving and consumption. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the motion of the global economy. Section 4 carries out comparative dynamic analysis to examine the impact of changes in some parameters on the motion of the global economy. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. THE MODEL

The model is developed within the framework of the neoclassical growth theory with international trade. Most neoclassical growth models are based on the pioneering works of Solow (1956). The Solow model has been extended and generalized in many studies (e.g., Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). Our model is specially based on the Uzawa two sector growth model (Uzawa, 1961, 1963). Our model is based on the basic features of other two approaches in the literature of neoclassical growth theory with trade and the growth model with public debt of Diamond (1965). The household decision is based on Zhang's approach (Zhang, 1993, 2005). We consider a world economy which consists of any number of countries, indexed by $j = 1, ..., J$. Country j has a fixed population, $\bar{N}_j$, ($j = 1, ..., J$). Each national economy consists of three sectors. We call them respectively tradable sector, non-tradable sector, and public sector. As in Ikeda and Ono (1992), We assume that all the national economies can produce a homogenous tradable commodity. The commodity is like the commodity in the Solow model which can be consumed and invested. Each national economy can thus produce one (durable) good in the global economy, one non-tradable (national) good, and one public services. Households own assets and distribute their incomes to consume and to save. The three sectors use capital and labor. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. The public sector uses capital and labor as inputs and supplies public services which are freely available to consumers. The public sector is financially by the government which taxes the household and the two production sectors and has debt.

We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th country. Capital depreciates at a constant exponential rate δ_j in country j, being independent of the manner of use within each country. Let $p_j(t)$ denote the price of non-tradable good. We use subscript index, i, s and p to stand for, respectively, tradable good sector, non-tradable good sector, and public sector in country j. We use $N_{jm}(t)$ and $K_{jm}(t)$ to stand for the labor force and capital stocks employed by sector m in country j. Let $F_{jm}(t)$ stand for the output level of sector m in country j. We use τ_{ji}, τ_{js}, τ_{jw}, and τ_{ja}, stand for, respectively, the fixed tax rates on the output of the tradeable sector, the output of the non-tradeable sector, the wage income, and the interest income. We define $\bar{\tau}_{jx} \equiv 1 - \tau_{jx}$, $x = i, s, w, k$.

The labor supply

The aggregated labor force $N_j(t)$ of country j is given by

$$N_j = h_j \bar{N}_j, \tag{1}$$

where h_j is the level of human capital in country j.

Production functions

We assume that production of sector (j, q) is to combine 'qualified labor force', $N_{jq}(t)$, and physical capital, $K_{jq}(t)$. We use the conventional production function to describe the relationship between inputs and output. The production process is described by

$$F_{jq}(t) = A_{jq} K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \ \ A_{jq}, \alpha_{jq}, \beta_{jq} > 0, \ \ \alpha_{jq} + \beta_{jq} = 1, \tag{2}$$

where A_{jq}, α_{jq}, and β_{jq} are positive parameters. The production functions are neoclassical. They are homogeneous of degree one with the inputs. In this study, we assume that levels of human capital are exogenous and total factor productivities are fixed.

Marginal conditions

Each production sector chooses capital and labor to maximize its profit. The marginal conditions are

$$r(t)+\delta_j = \frac{\alpha_{ji}\,\bar{\tau}_{ji}\,F_{ji}(t)}{K_{ji}(t)}, \quad w_j(t) = \frac{\beta_{ji}\,\bar{\tau}_{ji}\,F_{ji}(t)}{N_{ji}(t)}, \tag{3}$$

$$r(t)+\delta_j = \frac{\alpha_{js}\,\bar{\tau}_{js}\,p_{js}(t)F_{js}(t)}{K_{js}(t)}, \quad w_j(t) = \frac{\beta_{js}\,\bar{\tau}_{js}\,p_{js}(t)F_{js}(t)}{N_{js}(t)}, \tag{4}$$

where δ_j is depreciation rate of physical capital in country j.

The public sector

The production of public services is to combine capital $K_{jp}(t)$ and labor force $N_{jp}(t)$ as follows

$$F_{jp}(t) = A_{jp}\,K_{jp}^{\alpha_{0jp}}(t)N_{jp}^{\beta_{0jp}}(t), \quad \alpha_{0jp}, \beta_{0jp}, A_{jp} > 0. \tag{5}$$

Let $Y_{jp}(t)$ stand for government 's expenditure on supplying the public goods and services. We define the national output by

$$Y_j(t) = F_{ij}(t) + p_{js}(t)F_{js}(t).$$

This study assumes that $Y_p(t)$ is proportional to the national output as follows

$$Y_{jp}(t) = \tau_j\,Y_j(t), \tag{6}$$

where $\tau_j\,(0<\tau_j<1)$. is a non-negative parameter. This implies that the government expenditure is endogenously determined. The public sector is faced with the following budget constraint

$$(r(t)+\delta_j)K_{jp}(t) + w_j(t)N_{jp}(t) = Y_{jp}(t). \tag{7}$$

Maximization of public services under the budget constraint yields

$$\left(r(t)+\delta_j\right)K_{jp}(t)=\alpha_{jp}\,Y_{jp}(t),\ \ w_j(t)N_{jp}(t)=\beta_{jp}\,Y_{jp}(t), \tag{8}$$

in which

$$\alpha_{jp}\equiv\frac{\alpha_{0jp}}{\alpha_{0jp}+\beta_{0jp}},\ \ \beta_{jp}\equiv\frac{\beta_{0jp}}{\alpha_{0jp}+\beta_{0jp}}.$$

The current and disposable incomes of domestic households

We model behavior of domestic households by Zhang's approach (Zhang, 1993). The implications of this approach are similar to those in the Keynesian consumption function and models based on the permanent income hypothesis, which are empirically much more valid than the approaches in the Solow model or the in Ramsey model (Zhang, 2005). First, we use $a_j(t)$ to represent the value of wealth owned by a representative household. The wealth gets return rate $r(t)$. Both wage income and income from wealth are taxed by the government. The current income $y_j(t)$ is

$$y_j(t)=\bar{\tau}_{jw}\,h_j\,w_j(t)+\bar{\tau}_{ja}\,r(t)a_j(t). \tag{9}$$

The disposable income at any point in time is the sum of the current income and the value of wealth. We have the disposable income $\hat{y}_j(t)$ as follows

$$\hat{y}_j(t)=y_j(t)+a_j(t). \tag{10}$$

The disposable income is used for saving and consumption. At time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between consuming and saving.

The budget of domestic households

At each point in time the household distributes the total available budget between consumption of non-tradable good $c_{js}(t)$, tradable good $c_{ji}(t)$, and saving $s_j(t)$. The budget constraint is

$$\left(1+\tilde{\tau}_{js}\right)p_j(t)c_s(t)+\left(1+\tilde{\tau}_{ji}\right)c_{ji}(t)+s_j(t)=\hat{y}_j(t). \tag{11}$$

Equation (11) means that the consumption and saving exhaust the consumers' disposable income.

The utility functions

We assume that utility level $U_j(t)$ of the household is dependent on $c_{js}(t)$, $c_{ji}(t)$ and $s_j(t)$ as follows

$$U_j(t)=\theta_j\left(F_{jp}(t)\right)c_{js}^{\gamma_{0j}}(t)c_{ji}^{\xi_{0j}}(t)s_j^{\lambda_{0j}},\ \ \gamma_{0j},\xi_{0j},\lambda_{0j}>0,$$

in which γ_{0j}, ξ_{0j}, and λ_{0j} are the elasticities of utility with regard to non-tradable good, tradable good, and saving. We call γ_{0j}, ξ_{0j}, and λ_{0j} propensities to consume non-tradable good, to consume tradable good, and to hold wealth, respectively. We note that $\theta_j\left(F_{jp}(t)\right)$ takes account of possible impact of public services on the utility. Maximizing $U_j(t)$ subject to (11) yields

$$c_{js}(t)=\frac{\gamma_j\,\hat{y}_j(t)}{p_j(t)},\ \ c_{ji}(t)=\xi_j\,\hat{y}_j(t),\ \ s_j(t)=\lambda_j\,\hat{y}_j(t), \tag{12}$$

where

$$\gamma_j\equiv\frac{\rho_j\,\gamma_{j0}}{1+\tilde{\tau}_{js}},\ \ \xi_j\equiv\frac{\rho_j\,\xi_{j0}}{1+\tilde{\tau}_{ji}},\ \ \lambda_j\equiv\rho_j\,\lambda_{j0},\ \ \rho_j\equiv\frac{1}{\gamma_{j0}+\xi_{j0}+\lambda_{j0}}.$$

The change in wealth

According to the definition of $s_j(t)$, the wealth accumulation of the household is

$$\dot{a}_j(t) = s_j(t) - \dot{a}_j(t). \tag{13}$$

This equation states that the change in wealth equals the saving minus the dissaving.

The government budget

The government finances current spending by collecting taxes and issuing interest-bearing debt. The income comes from taxing the two sectors, the interest income of wealth, and consumption. Let $T_p(t)$ stand for the government's tax income. We have

$$T_{jp}(t) = \tau_{ji} F_{ji}(t) + \tau_{js} p_j(t) F_{js}(t) + \left[\tau_{ja} r(t) a_j(t) + \tilde{\tau}_{ji} c_{ji}(t) + \tilde{\tau}_{js} p_j(t) c_{js}(t)\right] \bar{N}_j . \tag{14}$$

The dynamics of governments' debts

The governments' debts can be owned by domestic as well as foreign households. The rate of interest on debts is determined in the global market. Country j's government debt follows the following dynamics

$$\dot{D}_j(t) = r(t) D_j(t) + Y_{jp}(t) - T_{jp}(t). \tag{15}$$

Market clearing in non-tradable good markets

The demand for non-tradable good equals the supply at any point time in each country

$$c_{js}(t) \bar{N}_j = F_{js}(t). \tag{16}$$

National capital stock is fully employed

We use $F_j(t)$ to stand for the capital stock employed by the national economy. The national capital stock is fully employed

$$K_{ji}(t)+K_{js}(t)+K_{jp}(t)=K_j(t), \quad j=1,\ldots,J. \tag{17}$$

Full employment of the labor force

We assume that the labor force is fully employed

$$N_{ji}(t)+N_{js}(t)+N_{jp}(t)=N_j. \tag{18}$$

Market clearing in tradable good markets

The global capital stock $K(t)$ is equal to the total capital employed by all the countries. That is

$$K(t)=\sum_{j=1}^{J}K_j(t). \tag{19}$$

The world production is equal to the world consumption and world net savings. That is

$$C(t)+S(t)-K(t)+\sum_{j=1}^{J}\delta_j K_j(t)=F(t),$$

where

$$C(t)\equiv\sum_{j=1}^{J}c_j(t)\bar{N}_j, \quad S(t)\equiv\sum_{j=1}^{J}s_j(t)\bar{N}_j, \quad F(t)\equiv\sum_{j=1}^{J}F_{ji}(t).$$

Global wealth balance

The wealth held by the households equals the global wealth

$$\sum_{j=1}^{N}a_j(t)\bar{N}_j=K(t)+\sum_{j=1}^{N}D_j(t). \tag{20}$$

The national debt

According to the concepts, we have the national debt $\bar{D}_j(t)$ as follows

$$\bar{D}_j(t) = D_j(t) + K_j(t) - a_j(t)\bar{N}_j. \tag{21}$$

We built the model with trade, economic growth, physical distribution, governments' debts, and national debts in the world economy in which the domestic markets of each country are perfectly competitive, international product, capital markets are freely mobile, and governments' policies on public good supplies are endogenous. The model synthesizes main ideas in economic growth theory and trade theory in a comprehensive framework. From a structural point of view, the model is general in the sense that some well-known models in economics can be considered as special cases. For instance, if the countries are identical, and human capital is constant, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961, 1963). Our model is also structurally similar to the Oniki-Uzawa trade model (Oniki and Uzawa, 1965). It is influenced by the Diamond model (Diamond, 1965).

3. THE DYNAMICS AND EQUILIBRIUM

The economic system contains many variables. These variables are nonlinearly related. For illustration, the rest of the study simulates the model. In order to simulate the model with computer, we provide a computational procedure so that one can easily follow the motion of the economic system with any set of parameters and initial conditions. In the appendix, we show that the dynamics of the economy can be expressed as J differential equations. First, we introduce a variable $z_1(t)$ by

$$z_1(t) \equiv \frac{r(t) + \delta_j}{w_1(t)}.$$

We now show that the dynamics can be expressed by differential equations with $z_1(t)$, $\{a_j(t)\}$ $= (a_2(t), ..., a_J(t))$ and $\{D_j(t)\} = (D_1(t), ..., D_J(t))$ as the variables.

Lemma

The motion of J variables, $z_1(t)$, $\{a_j(t)\}$ and $\{D_j(t)\}$ is given by the following $2J$ differential equations

$$\dot{z}_1(t) = \Lambda_1(z_1(t), \{a_j(t), (D_j)\}),$$
$$\dot{a}_j(t) = \Lambda_j(z_1(t), \{a_j(t)\}, (D_j)), \quad j = 2, ..., J,$$
$$\dot{D}_j(t) = \Psi_j(z_1(t), \{a_j(t)\}, (D_j(t))), \quad j = 1, ..., J, \tag{22}$$

where $\Lambda_j(t)$ and $\Psi_j(t)$ are functions of $z_1(t)$, $\{a_j(t)\}$ and $\{D_j(t)\}$ defined in the appendix. The values of the other variables are given as functions of $z_1(t)$, $\{a_j(t)\}$ and $\{D_j(t)\}$ at any point in time by the following procedure: by (A2) $r(t)$ by (A2) $\to$ $z_j(t)$ by (A3) $\to$ $w_j(t)$ by (A4) $\to$ $p_j(t)$ by (A5) $\to$ $a_1(t)$ by (A18) $\to$ $N_{js}(t)$ by (A8) $\to$ $N_{ji}(t)$ by (A13) $\to$ $N_{jp}(t)$ by (A14) $\to$ $K_{jp}(t)$, $K_{ji}(t)$ and $K_{js}(t)$ by (A1) $\to$ $K_j(t)$ by (A16) $\to$ $K(t)$ by (A17) $\to$ $\hat{y}_j(t)$ by (A6) $\to$ $F_{jq}(t)$ by (1) $\to$ $c_j(t)$, $c_{js}(t)$, and $s_j(t)$ by (10) $\to$ $Y_{jq}(t)$ by (A10) $\to$ $T_{jq}(t)$ by (14).

The lemma implies that the motion of economic system at any point in time can be identified by following the lemma with initial conditions. For simulation, we specify values of the parameters. We consider the world consists of three national economies, i.e., $J = 3$. We specify the parameter values as follows

$$N_1 = 130,\ N_2 = 500,\ N_3 = 500,\ h_1 = 8,\ h_2 = 3,\ h_3 = 1,\ \delta_1 = 0.06,\ \delta_2 = \delta_3 = 0.05,\ \alpha_{jp} = 0.3,$$
$$\beta_{jp} = 0.7,\ \alpha_{1i} = 0.3,\ \alpha_{2i} = 0.31,\ \alpha_{3i} = 0.32,\ \alpha_{1s} = 0.35,\ \alpha_{2s} = 0.34,\ \alpha_{3s} = 0.33,\ A_{1i} = 1.2,$$
$$A_{2i} = 1.1,\ A_{3i} = 1,\ A_{1s} = 1.1,\ A_{2s} = 1,\ A_{3s} = 0.9,\ A_{1p} = 1,\ A_{2p} = 0.9,\ A_{3p} = 0.8,\ \lambda_{01} = 0.8,$$
$$\xi_{01} = 0.1,\ \gamma_{01} = 0.06,\ \lambda_{02} = 0.7,\ \xi_{02} = 0.6,\ \gamma_{02} = 0.05,\ \lambda_{03} = 0.6,\ \xi_{03} = 0.06,\ \gamma_{03} = 0.05,$$
$$\tau_j = 0.02,\ \tau_{ji} = \tau_{js} = \tau_{jw} = \bar{\tau}_{ja} = \tilde{\tau}_{ji} = \tilde{\tau}_{js} = 0.01,\ \tau_2 = 0.04,\ \tau_3 = 0.03. \tag{23}$$

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Country 1, 2 and 3's populations are respectively 130, 500 and 500. Country 1 has the smallest population. Country 1 has the highest human capital and Country 2 next. The physical capital depreciation rates of the three economies are approximately 0.05. The total factor productivities are different between three economies. Country 1 's total factor productivity is highest and Country 3 's total factor productivity is lowest. The output elasticities with respect labor and capital also vary between countries. We specify the values of the parameters, α_{ji} and α_{js} in the Cobb-Douglas productions approximately equal to 0.3. The household preferences of the three economies also vary. As we already provided the procedure to follow the motion of each variable in the system, it is straightforward to plot the motion with computer. We specify the initial conditions as follows

$$z_1(0) = 0.11, \quad a_2(0) = 3.2, \quad a_3(0) = 10, \quad D_1(0) = 142, \quad D_2(0) = 123, \quad D_3(0) = 112.$$

The motion of the system is given in Figure 1. In the figure the national outputs $Y_j(t)$ are defined as follows

$$Y_j \equiv F_{ji} + p_{js} F_{js} .$$

As shown in Figure 1, different countries will not experience convergence in per capita income, consumption and wealth in the long term. There are extensive discussions about income and wealth convergence between nations in the literature of economic growth and development. The literature on global economic growth provides little insights into the issues as most of these studies are based on the insights from analyzing models of closed economies (Barro and Sala-i-Martin, 1995, Barro, 2013). As economics lacks analytical frameworks for analyzing global growth and trades with microeconomic foundation, theoretical economics fails to properly discuss issues related to global income and wealth convergence. For instance, the conclusions from the Solow model for closed economies are often used to discuss issues related to income inequalities between countries.

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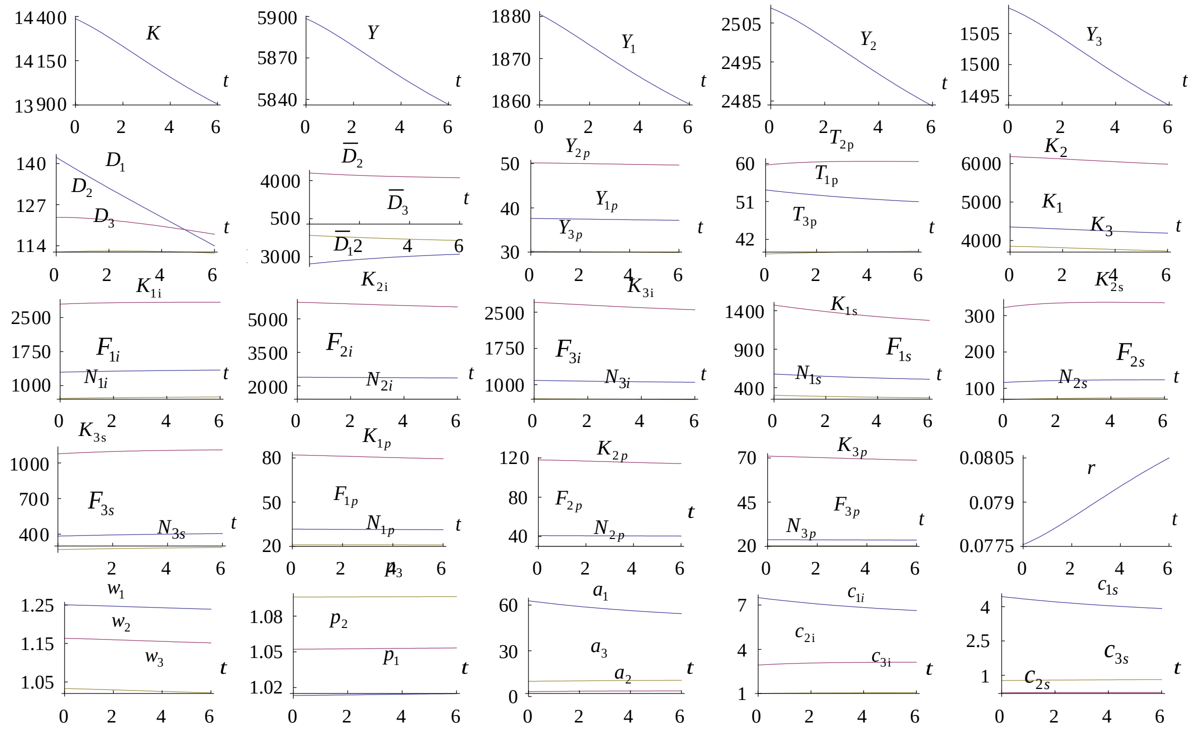

Figure 1. The Motion of the Global Economy

Following the lemma and (23), we calculate the equilibrium values of the variables as follows

$$K = 13193.9, \quad Y = 5742.3, \quad r = 0.085,$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.22 \\ 1.13 \\ 1.01 \end{pmatrix}, \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.02 \\ 1.06 \\ 1.1 \end{pmatrix}, \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 1830 \\ 2444 \\ 1469 \end{pmatrix}, \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} 144.2 \\ 126.4 \\ 116.1 \end{pmatrix}, \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \end{pmatrix} = \begin{pmatrix} -2195.5 \\ 3993.2 \\ -1797.7 \end{pmatrix},$$

$$\begin{pmatrix} Y_{1p} \\ Y_{2p} \\ Y_{3p} \end{pmatrix} = \begin{pmatrix} 36.6 \\ 48.9 \\ 29.4 \end{pmatrix}, \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 3976.2 \\ 5680.7 \\ 3537 \end{pmatrix}, \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 1360.9 \\ 2315.6 \\ 1018.8 \end{pmatrix}, \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 772 \\ 1396 \\ 683 \end{pmatrix}, \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 2782 \\ 5253 \\ 2386 \end{pmatrix},$$

$$\begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 461 \\ 1121.6 \\ 410 \end{pmatrix}, \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 247.1 \\ 74 \\ 296.9 \end{pmatrix}, \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 1119 \\ 319 \\ 1086 \end{pmatrix}, \begin{pmatrix} F_{1p} \\ F_{2p} \\ F_{3p} \end{pmatrix} = \begin{pmatrix} 30.8 \\ 39.9 \\ 23.2 \end{pmatrix},$$

$$\begin{pmatrix} N_{1p} \\ N_{2p} \\ N_{3p} \end{pmatrix} = \begin{pmatrix} 21 \\ 30.2 \\ 20.5 \end{pmatrix}, \begin{pmatrix} K_{1p} \\ K_{2p} \\ K_{3p} \end{pmatrix} = \begin{pmatrix} 75.6 \\ 108.4 \\ 65.1 \end{pmatrix}, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 48.6 \\ 3.63 \\ 10.9 \end{pmatrix}, \begin{pmatrix} c_{1i} \\ c_{2i} \\ c_{3i} \end{pmatrix} = \begin{pmatrix} 6.01 \\ 3.08 \\ 1.08 \end{pmatrix}, \begin{pmatrix} c_{1s} \\ c_{2s} \\ c_{3s} \end{pmatrix} = \begin{pmatrix} 3.55 \\ 0.24 \\ 0.81 \end{pmatrix}.$$

It is straightforward to calculate the six eigenvalues as follows

$$\{-0.463, -0.161, -0.116, \ 0.088, \ 0.085, \ 0.085\}.$$

This implies that the world economy is unstable. The instability means that we cannot effectively conduct comparative statics analysis and comparative dynamic analysis in the long run as the system will not move along a stable path over time. We conduct comparative dynamic analysis in the short run.

4. SHORT-RUN COMPARATIVE DYNAMIC ANALYSIS

It is important to ask questions such as how a change in one country's conditions affects the national economy and global economies. For instance, if a country changes its policy to supply public good, the global economic growth, governments' debts and national debts may be affected over time. We can easily answer the question as we can simulate the motion of the dynamic system. This section examines effects of changes in some parameters on the global economy. First, we introduce a variable $\overline{\Delta} x(t)$ to stand for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

4.1. Country 1's government increasing the ratio of expenditure to the national output

We now show effects of the following increase in country 1's ratio of expenditure to the national output: $\tau_1 : 0.02 \Rightarrow 0.03$. The simulation result is plotted in Figure 2. As the system contains many variables and these variables are connected to each other in nonlinear relations, it is

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difficult to verbally explain these relations over time. As the system is unstable, our simulation period is short-run. As country 1 spends more out of its national output, the global wealth, global income and national incomes of the three economies all fall. Government 1's debt rises and country 1's debt falls. The other two governments' debts are slightly increased. Country 3's debt rises and country 2's debt falls. Country 1 spends more on the public good. All the three countries' tax incomes and capital stocks employed by the economy are reduced over time. The wage rates, wealth levels, and consumption levels of tradable good and non-tradable in all the three national economies are reduced. The economic structural changes are plotted in Figure 2.

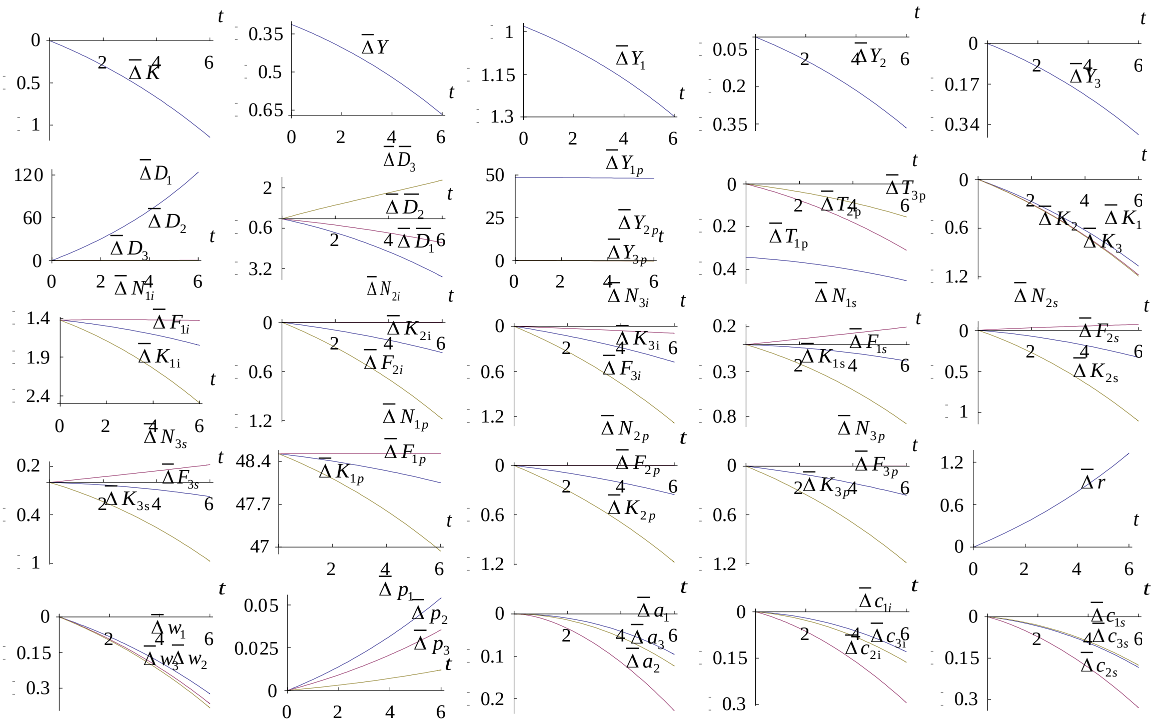

Figure 2. Country 1's Government Increasing the Ratio of Expenditure

The effects on the equilibrium point are listed as in (24). It can be seen that the effects on the equilibrium point are different from the short-term effects. At the new unstable equilibrium point government 1's debt is reduced and other two governments' debts are increased. The levels of per capita wealth and consumption are enhanced.

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$$\overline{\Delta} K = 2.28,\ \overline{\Delta} Y = 0.39,\ \overline{\Delta} r = -2.5,$$

$$\begin{pmatrix} \overline{\Delta} w_1 \\ \overline{\Delta} w_2 \\ \overline{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 0.64 \\ 0.72 \\ 0.75 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} p_1 \\ \overline{\Delta} p_2 \\ \overline{\Delta} p_3 \end{pmatrix} = \begin{pmatrix} -0.11 \\ -0.06 \\ -0.02 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} Y_1 \\ \overline{\Delta} Y_2 \\ \overline{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} -0.35 \\ -0.72 \\ -0.75 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} D_1 \\ \overline{\Delta} D_2 \\ \overline{\Delta} D_3 \end{pmatrix} = \begin{pmatrix} -148.5 \\ 2.92 \\ 2.1 \end{pmatrix},$$

$$\begin{pmatrix} \overline{\Delta} \overline{D}_1 \\ \overline{\Delta} \overline{D}_2 \\ \overline{\Delta} \overline{D}_3 \end{pmatrix} = \begin{pmatrix} 7.7 \\ 3.1 \\ -2.54 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} Y_{1p} \\ \overline{\Delta} Y_{2p} \\ \overline{\Delta} Y_{3p} \end{pmatrix} = \begin{pmatrix} 49.5 \\ 0.72 \\ 0.75 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} K_1 \\ \overline{\Delta} K_2 \\ \overline{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} 2.1 \\ 2.3 \\ 2.4 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} F_{1i} \\ \overline{\Delta} F_{2i} \\ \overline{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} -0.7 \\ 0.7 \\ 0.8 \end{pmatrix},$$

$$\begin{pmatrix} \overline{\Delta} N_{1i} \\ \overline{\Delta} N_{2i} \\ \overline{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} -1.32 \\ 0 \\ 0.01 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} K_{1i} \\ \overline{\Delta} K_{2i} \\ \overline{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 2.3 \\ 2.7 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} F_{1s} \\ \overline{\Delta} F_{2s} \\ \overline{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.8 \\ 0.8 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} N_{1s} \\ \overline{\Delta} N_{2s} \\ \overline{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0 \\ -0.01 \end{pmatrix},$$

$$\begin{pmatrix} \overline{\Delta} K_{1s} \\ \overline{\Delta} K_{2s} \\ \overline{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} 2.1 \\ 2.3 \\ 2.4 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} F_{1p} \\ \overline{\Delta} F_{2p} \\ \overline{\Delta} F_{3p} \end{pmatrix} = \begin{pmatrix} 49.5 \\ 0.69 \\ 0.7 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} N_{1p} \\ \overline{\Delta} N_{2p} \\ \overline{\Delta} N_{3p} \end{pmatrix} = \begin{pmatrix} 48.5 \\ 0 \\ 0 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} K_{1p} \\ \overline{\Delta} K_{2p} \\ \overline{\Delta} K_{3p} \end{pmatrix} = \begin{pmatrix} 51.7 \\ 2.3 \\ 2.4 \end{pmatrix},$$

$$\begin{pmatrix} \overline{\Delta} a_1 \\ \overline{\Delta} a_2 \\ \overline{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} \overline{\Delta} c_{1i} \\ \overline{\Delta} c_{2i} \\ \overline{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} 0.63 \\ 0.71 \\ 0.74 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} c_{1s} \\ \overline{\Delta} c_{2s} \\ \overline{\Delta} c_{3s} \end{pmatrix} = \begin{pmatrix} 0.73 \\ 0.78 \\ 0.76 \end{pmatrix}. \tag{24}$$

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A 4.2. A rise in country 1's human capital

We study effects of a rise in country 1's human capital on the global economy. Figure 3 plots what happen to the global economy when the human capital is enhanced as follows: $h_1 : 8 \Rightarrow 8.2$. In the short term, a rise in the human capital not only reduces government 1's debt, but also brings down the other two governments' debts. The result is that the improved human capital increases the national outputs and thus tax incomes. Country 1's and country 2's national debts are increased and country 1's national debt is lowered. The public sectors in the three economies are expanded. All sectors produce more. All the household have more wealth and consume more tradable and non-tradable goods. The beneficial results are associated with falling in the rate of interest and rising in the wage rates. The lowering rate of interest reduces the government financial burdens and rising wage rates increase the governments' tax income.

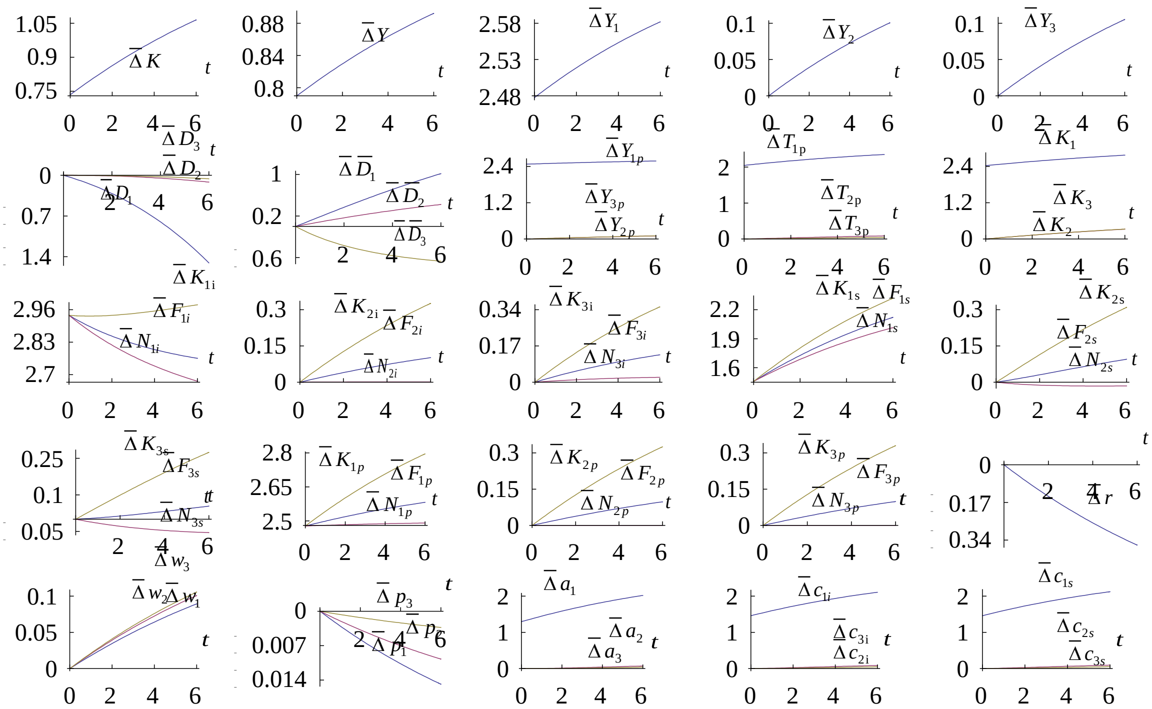

Figure 3. A Rise in Country 1's Human Capital

The shift of the equilibrium point is listed in (25).

$$\overline{\Delta}K = 1.33,\ \overline{\Delta}Y = 0.96,\ \overline{\Delta}r = -0.64,$$

$$\begin{pmatrix}\overline{\Delta}w_1\\ \overline{\Delta}w_2\\ \overline{\Delta}w_3\end{pmatrix} = \begin{pmatrix}0.16\\0.18\\0.19\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}p_1\\ \overline{\Delta}p_2\\ \overline{\Delta}p_3\end{pmatrix} = \begin{pmatrix}-0.03\\-0.02\\-0.01\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}Y_1\\ \overline{\Delta}Y_2\\ \overline{\Delta}Y_3\end{pmatrix} = \begin{pmatrix}2.67\\0.18\\0.19\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}D_1\\ \overline{\Delta}D_2\\ \overline{\Delta}D_3\end{pmatrix} = \begin{pmatrix}3.03\\0.73\\0.53\end{pmatrix},$$

$$\begin{pmatrix}\overline{\Delta}\overline{D}_1\\ \overline{\Delta}\overline{D}_2\\ \overline{\Delta}\overline{D}_3\end{pmatrix} = \begin{pmatrix}1.93\\0.78\\-0.64\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}Y_{1p}\\ \overline{\Delta}Y_{2p}\\ \overline{\Delta}Y_{3p}\end{pmatrix} = \begin{pmatrix}2,67\\0.18\\0.19\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}K_1\\ \overline{\Delta}K_2\\ \overline{\Delta}K_3\end{pmatrix} = \begin{pmatrix}3.05\\0.59\\0.6\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}F_{1i}\\ \overline{\Delta}F_{2i}\\ \overline{\Delta}F_{3i}\end{pmatrix} = \begin{pmatrix}2.67\\0.18\\0.19\end{pmatrix},$$

$$\begin{pmatrix}\overline{\Delta}N_{1i}\\ \overline{\Delta}N_{2i}\\ \overline{\Delta}N_{3i}\end{pmatrix} = \begin{pmatrix}2.5\\0\\0\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}K_{1i}\\ \overline{\Delta}K_{2i}\\ \overline{\Delta}K_{3i}\end{pmatrix} = \begin{pmatrix}3.05\\0.6\\0.6\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}F_{1s}\\ \overline{\Delta}F_{2s}\\ \overline{\Delta}F_{3s}\end{pmatrix} = \begin{pmatrix}2.69\\0.2\\0.19\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}N_{1s}\\ \overline{\Delta}N_{2s}\\ \overline{\Delta}N_{3s}\end{pmatrix} = \begin{pmatrix}2.49\\0\\0\end{pmatrix},$$

$$\begin{pmatrix}\overline{\Delta}K_{1s}\\ \overline{\Delta}K_{2s}\\ \overline{\Delta}K_{3s}\end{pmatrix} = \begin{pmatrix}3.05\\0.59\\0.59\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}F_{1p}\\ \overline{\Delta}F_{2p}\\ \overline{\Delta}F_{3p}\end{pmatrix} = \begin{pmatrix}2.67\\0.18\\0.18\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}N_{1p}\\ \overline{\Delta}N_{2p}\\ \overline{\Delta}N_{3p}\end{pmatrix} = \begin{pmatrix}2.5\\0\\0\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}K_{1p}\\ \overline{\Delta}K_{2p}\\ \overline{\Delta}K_{3p}\end{pmatrix} = \begin{pmatrix}3.05\\0.59\\0.6\end{pmatrix},$$

$$\begin{pmatrix}\overline{\Delta}a_1\\ \overline{\Delta}a_2\\ \overline{\Delta}a_3\end{pmatrix} = \begin{pmatrix}\overline{\Delta}c_{1i}\\ \overline{\Delta}c_{2i}\\ \overline{\Delta}c_{3i}\end{pmatrix} = \begin{pmatrix}2.66\\0.18\\0.19\end{pmatrix},\ \begin{pmatrix}\overline{\Delta}c_{1s}\\ \overline{\Delta}c_{2s}\\ \overline{\Delta}c_{3s}\end{pmatrix} = \begin{pmatrix}2.7\\0.2\\0.19\end{pmatrix}. \tag{25}$$

4.3. A rise in the total factor productivity of country 1's tradable sector

We study effects of a rise in country 1's tradable sector on the global economy. It has been argued that productivity differences explain much of the variation in incomes across countries, and technology plays a key role in determining productivity. We see what will happen to the global economy when country 1's total factor productivity is enhanced as follows: $A_{1i}: 1.2 \Rightarrow 1.25$. The simulation result is plotted in Figure 4. As the technology is improved, country 1's tradable sector increases its output and two inputs. The wage rate in country is increased and the wage rates in the other two economies are reduced. All the households own less wealth and consume less tradable goods. The rate of interest is enhanced. The global capital stock falls. The global income and country 1's national output level are increased, while the other two countries' national output are reduced. All the governments' debts are enhanced. Countr3's national debt is augmented, while the other two countries' national debts fall. Country 1's public good supply and tax income are increased, while the corresponding variables of the other two countries are reduced. The price of non-tradable good in country 1 falls, while the prices in the other two countries are slightly affected.

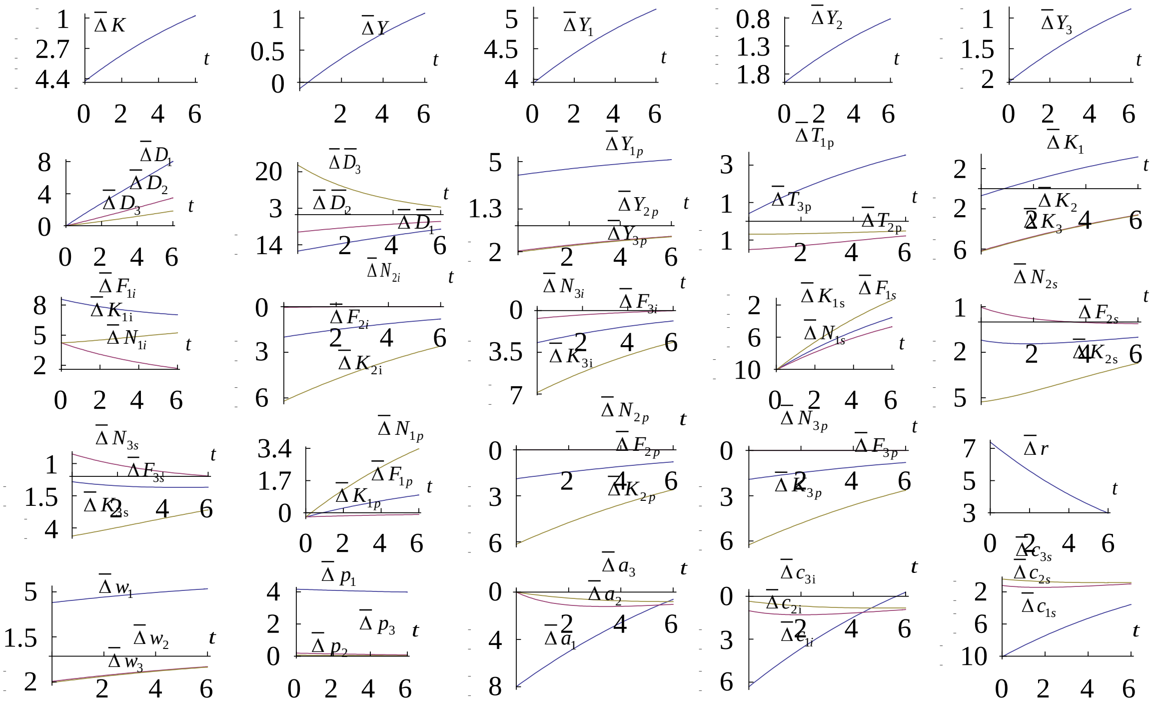

Figure 4. A Rise in the Total Factor Productivity of Country 1's Tradable Sector

The shift of the equilibrium point is listed in (26).

$$\bar{\Delta} K = 3.2, \quad \bar{\Delta} Y = 2.3, \quad \bar{\Delta} r = -1.5,$$

$$\begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 6.4 \\ 0.43 \\ 0.45 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_1 \\ \bar{\Delta} p_2 \\ \bar{\Delta} p_3 \end{pmatrix} = \begin{pmatrix} 3.78 \\ -0.04 \\ -0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} 6.4 \\ 0.43 \\ 0.45 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} D_1 \\ \bar{\Delta} D_2 \\ \bar{\Delta} D_3 \end{pmatrix} = \begin{pmatrix} 7.32 \\ 1.75 \\ 1.26 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} \bar{D}_1 \\ \bar{\Delta} \bar{D}_2 \\ \bar{\Delta} \bar{D}_3 \end{pmatrix} = \begin{pmatrix} 4.61 \\ 1.85 \\ -1.52 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} Y_{1p} \\ \bar{\Delta} Y_{2p} \\ \bar{\Delta} Y_{3p} \end{pmatrix} = \begin{pmatrix} 6.41 \\ 0.42 \\ 0.45 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} 7.36 \\ 1.4 \\ 1.4 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} 6.4 \\ 0.43 \\ 0.46 \end{pmatrix}, \tag{26}$$

$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} 0.002 \\ 0 \\ 0.003 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} 7.37 \\ 1.4 \\ 1.4 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} 2.51 \\ 0.47 \\ 0.46 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0 \\ -0.01 \end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta} K_{1s}\\ \bar{\Delta} K_{2s}\\ \bar{\Delta} K_{3s}\end{pmatrix}=\begin{pmatrix}7.4\\ 1.4\\ 1.4\end{pmatrix},\ \begin{pmatrix}\bar{\Delta} F_{1p}\\ \bar{\Delta} F_{2p}\\ \bar{\Delta} F_{3p}\end{pmatrix}=\begin{pmatrix}2.2\\ 0.4\\ 0.4\end{pmatrix},\ \begin{pmatrix}\bar{\Delta} N_{1p}\\ \bar{\Delta} N_{2p}\\ \bar{\Delta} N_{3p}\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix},\ \begin{pmatrix}\bar{\Delta} K_{1p}\\ \bar{\Delta} K_{2p}\\ \bar{\Delta} K_{3p}\end{pmatrix}=\begin{pmatrix}7.4\\ 1.4\\ 1.4\end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta} a_{1}\\ \bar{\Delta} a_{2}\\ \bar{\Delta} a_{3}\end{pmatrix}=\begin{pmatrix}6.4\\ 0.4\\ 0.4\end{pmatrix},\ \begin{pmatrix}\bar{\Delta} c_{1i}\\ \bar{\Delta} c_{2i}\\ \bar{\Delta} c_{3i}\end{pmatrix}=\begin{pmatrix}6.4\\ 0.4\\ 0.4\end{pmatrix},\ \begin{pmatrix}\bar{\Delta} c_{1s}\\ \bar{\Delta} c_{2s}\\ \bar{\Delta} c_{3s}\end{pmatrix}=\begin{pmatrix}2.5\\ 0.5\\ 0.5\end{pmatrix}.$$

4.4. The tax rate on country 1's tradable sector being enhanced

We now examine what happen to the global economy when government 1 increases its tax rate on the tradable sector as follows: $\tau_{1i}: 0.01 \Rightarrow 0.02$. The simulation result is plotted in Figure 5. As the tax rate is enhanced, the rate of interest falls. The governments have less financial burdens on paying debts. The global capital stock and income are increased. Country 1 changes slightly the capital stock, while the other two countries use more capital stocks. Country 1 has lower national output, while the other two countries have higher. Country 1's tax income and public good supply are augmented. The other two economies' tax incomes are slightly affected. The prices of non-tradable goods are reduced. Country 1's wage rate is reduced and the wage rates in the other two countries are enhanced.

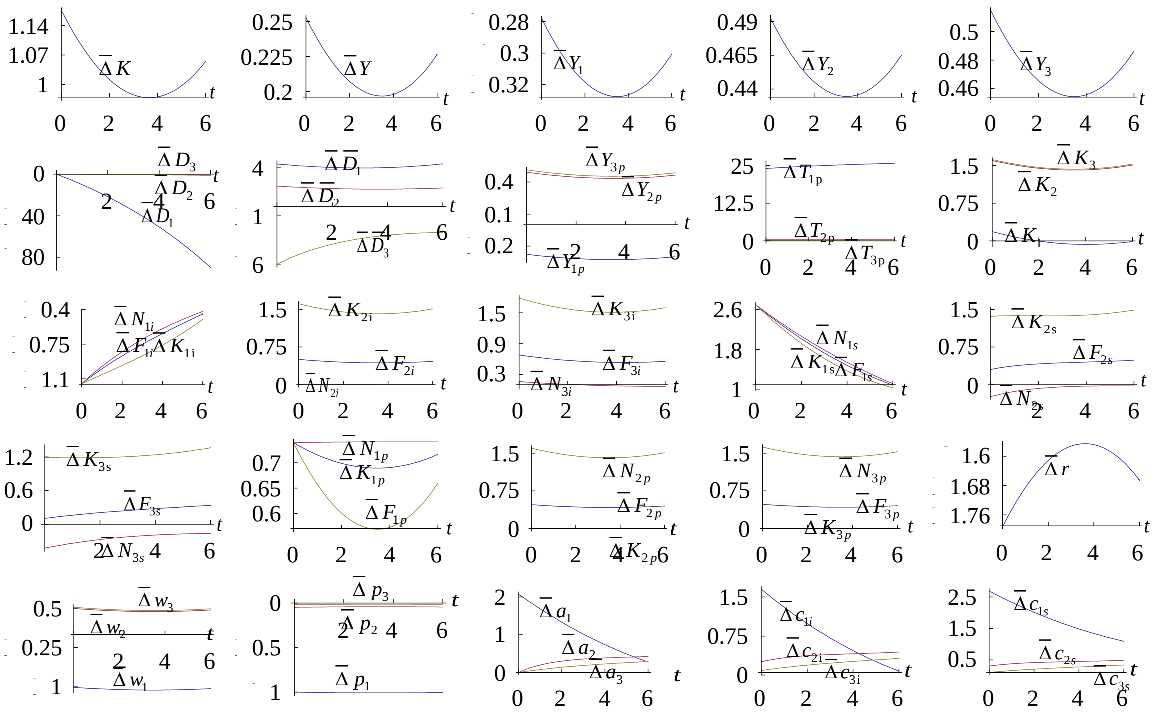

Figure 5. The Tax Rate on Country 1's Tradable Sector Being Enhanced

The shift of the equilibrium point is listed in (27).

$$\bar{\Delta} K = -2.4, \ \bar{\Delta} Y = -0.83, \ \bar{\Delta} r = 2.23,$$
$$\begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} -1.99 \\ -0.62 \\ -0.65 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} p_1 \\ \bar{\Delta} p_2 \\ \bar{\Delta} p_3 \end{pmatrix} = \begin{pmatrix} -0.85 \\ 0.06 \\ 0.02 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} -1.26 \\ -0.62 \\ -0.65 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} D_1 \\ \bar{\Delta} D_2 \\ \bar{\Delta} D_3 \end{pmatrix} = \begin{pmatrix} 102.9 \\ -2.5 \\ -1.8 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta} \bar{D}_1 \\ \bar{\Delta} \bar{D}_2 \\ \bar{\Delta} \bar{D}_3 \end{pmatrix} = \begin{pmatrix} -6.6 \\ -2.6 \\ 2.2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} Y_{1p} \\ \bar{\Delta} Y_{2p} \\ \bar{\Delta} Y_{3p} \end{pmatrix} = \begin{pmatrix} -1.26 \\ -0.62 \\ -0.65 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -3.25 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} -1 \\ -0.62 \\ -0.66 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} -0.02 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} -3.3 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -1.14 \\ -0.68 \\ -0.66 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0 \\ 0.01 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -3.24 \\ -2 \\ -2.02 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1p} \\ \bar{\Delta} F_{2p} \\ \bar{\Delta} F_{3p} \end{pmatrix} = \begin{pmatrix} -0.25 \\ -0.6 \\ -0.61 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_{1p} \\ \bar{\Delta} N_{2p} \\ \bar{\Delta} N_{3p} \end{pmatrix} = \begin{pmatrix} 0.74 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} K_{1p} \\ \bar{\Delta} K_{2p} \\ \bar{\Delta} K_{3p} \end{pmatrix} = \begin{pmatrix} -2.53 \\ -2 \\ -2.01 \end{pmatrix}, \quad (27)$$
$$\begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -1.98 \\ -0.62 \\ -0.64 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -1.98 \\ -0.62 \\ -0.64 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} = \begin{pmatrix} -1.14 \\ -0.68 \\ -0.66 \end{pmatrix}.$$

4.5. A rise in country 1's tax rate on non-tradable good

We now study effects of a rise in government 1's tax rate on the tradable sector as follows: $\tau_{1s} : 0.01 \Rightarrow 0.02$. The simulation result is plotted in Figure 6. As the tax rate is enhanced, the rate of interest falls. The governments have less financial burdens on paying debts. The global capital stock and income are increased. The countries have higher national output and use more capital stocks. Different from the effects of the rise in the tax rate on tradable good, country's 1 has higher national output partly because the price of tradable good in country 1 is enhanced. The other two countries have higher. Country 1's tax income and public good supply are augmented. The other two economies' tax incomes are slightly affected. The wage rates are enhanced.

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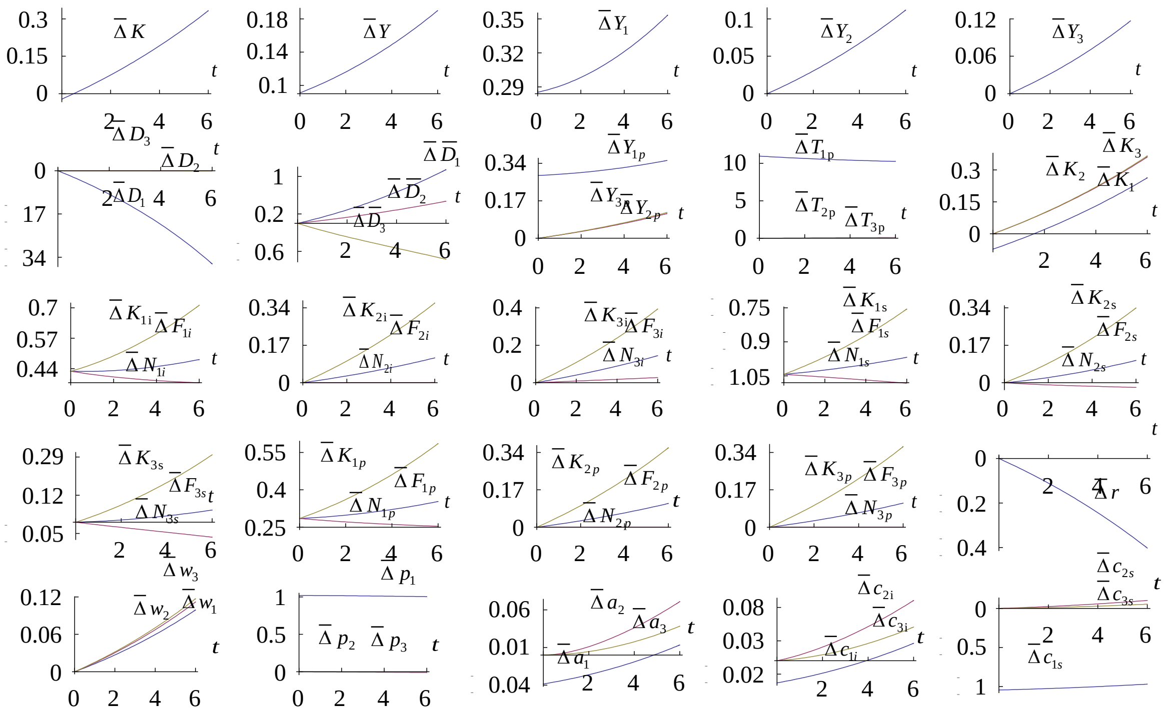

Figure 6. A Rise in Country 1's Tax Rate on Non-Tradable Good

The shift of the equilibrium point is listed in (28).

$$\bar{\Delta} K = -0.56,\ \ \bar{\Delta} Y = -0.09,\ \ \bar{\Delta} r = 0.61,$$

$$\begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.17 \\ -0.18 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} p_1 \\ \bar{\Delta} p_2 \\ \bar{\Delta} p_3 \end{pmatrix} = \begin{pmatrix} 1.05 \\ 0.02 \\ 0.01 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} 0.08 \\ -0.17 \\ -0.18 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} D_1 \\ \bar{\Delta} D_2 \\ \bar{\Delta} D_3 \end{pmatrix} = \begin{pmatrix} 37 \\ -0.69 \\ -0.5 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} \bar{D}_1 \\ \bar{\Delta} \bar{D}_2 \\ \bar{\Delta} \bar{D}_3 \end{pmatrix} = \begin{pmatrix} -1.83 \\ -0.74 \\ 0.6 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} Y_{1p} \\ \bar{\Delta} Y_{2p} \\ \bar{\Delta} Y_{3p} \end{pmatrix} = \begin{pmatrix} 0.08 \\ -0.17 \\ -0.18 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -0.57 \\ -0.56 \\ -0.57 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} 0.16 \\ -0.17 \\ -0.18 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0 \\ 0 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} -0.2 \\ -0.56 \\ -0.57 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -1.19 \\ -0.19 \\ -0.18 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -1.01 \\ 0 \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -1.51 \\ -0.56 \\ -0.56 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1p} \\ \bar{\Delta} F_{2p} \\ \bar{\Delta} F_{3p} \end{pmatrix} = \begin{pmatrix} 0.08 \\ -0.17 \\ -0.17 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_{1p} \\ \bar{\Delta} N_{2p} \\ \bar{\Delta} N_{3p} \end{pmatrix} = \begin{pmatrix} 0.24 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} K_{1p} \\ \bar{\Delta} K_{2p} \\ \bar{\Delta} K_{3p} \end{pmatrix} = \begin{pmatrix} -0.28 \\ -0.56 \\ -0.57 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.17 \\ -0.18 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.17 \\ -0.18 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} = \begin{pmatrix} -1.19 \\ -0.19 \\ -0.18 \end{pmatrix}. \quad (28)$$

4.6. Country 1 increasing the propensity to consume non-tradable good

One of common believes in encouraging one's country economic development is to consume more domestic goods. We now study what will happen to the global economy when country 1 increases the propensity to consume its domestic non-tradable product. We allow the propensity to consume the non-tradable good as follows: $\gamma_{10}: 0.06 \Rightarrow 0.07$. The simulation result is plotted in Figure 7. As the preference is changed, the representative household of country 1 holds less wealth, consumes more non-tradable good and less tradable good. The households in the other two economies have a little less wealth, and consume a little less two goods. The global wealth and output fall. Country 1's output rises initially and falls soon. The national outputs of the other two economies fall. Country 1's government debt is reduced, while other two countries' government debts are increased. The wage rates fall and the prices of non-tradable goods rise.

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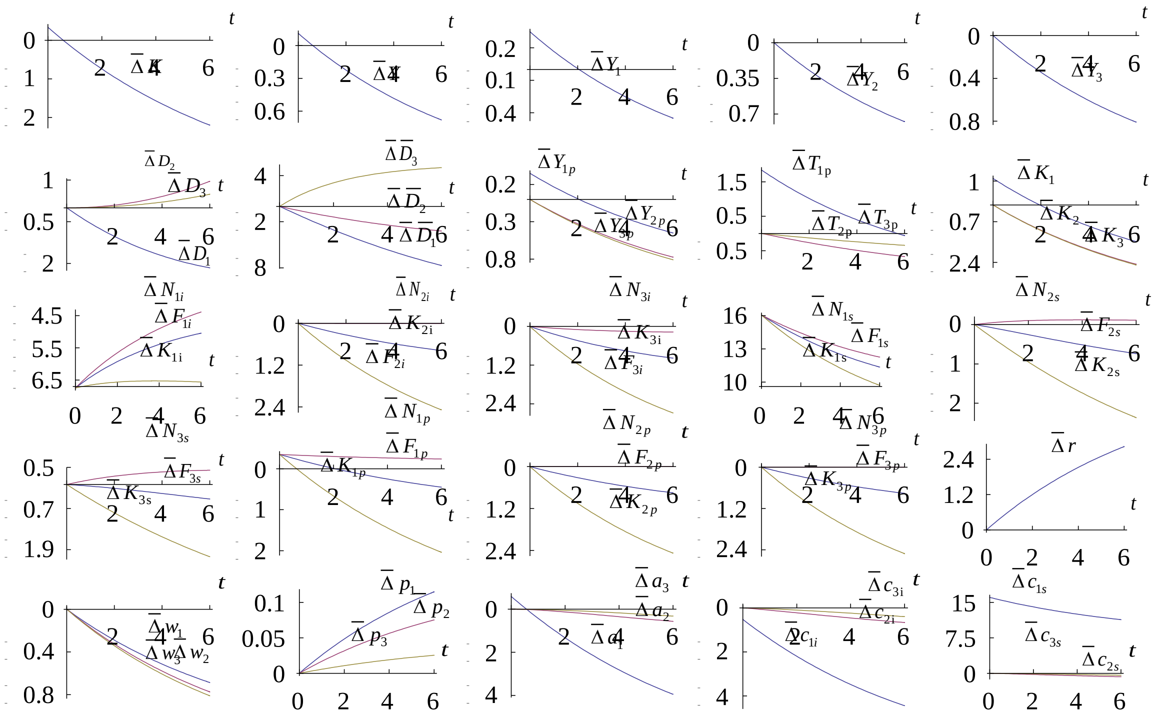

Figure 7. Country 1 Increasing the Propensity to Consume Non-Tradable Good

The shift of the equilibrium point is listed in (29).

$$\bar{\Delta} K = -3.93,\ \ \bar{\Delta} Y = -1.23,\ \ \bar{\Delta} r = 4.74,$$

$$\begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} -1.17 \\ -1.32 \\ -1.38 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} p_1 \\ \bar{\Delta} p_2 \\ \bar{\Delta} p_3 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.13 \\ 0.04 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1.32 \\ -1.38 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} D_1 \\ \bar{\Delta} D_2 \\ \bar{\Delta} D_3 \end{pmatrix} = \begin{pmatrix} -6.49 \\ -5.14 \\ -3.72 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} \bar{D}_1 \\ \bar{\Delta} \bar{D}_2 \\ \bar{\Delta} \bar{D}_3 \end{pmatrix} = \begin{pmatrix} -13.69 \\ -5.51 \\ 4.47 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} Y_{1p} \\ \bar{\Delta} Y_{2p} \\ \bar{\Delta} Y_{3p} \end{pmatrix} = \begin{pmatrix} -1 \\ -1.32 \\ -1.38 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -3.31 \\ -4.18 \\ -4.24 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} -4.27 \\ -1.32 \\ -1.39 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} -3.14 \\ 0 \\ -0.01 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} -6.87 \\ -4.18 \\ -4.25 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} 8.3 \\ -1.44 \\ -1.4 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} 9.8 \\ 0 \\ 0.02 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} 5.57 \\ -4.18 \\ -4.22 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1p} \\ \bar{\Delta} F_{2p} \\ \bar{\Delta} F_{3p} \end{pmatrix} = \begin{pmatrix} -1 \\ -1.27 \\ -1.29 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_{1p} \\ \bar{\Delta} N_{2p} \\ \bar{\Delta} N_{3p} \end{pmatrix} = \begin{pmatrix} 0.18 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} K_{1p} \\ \bar{\Delta} K_{2p} \\ \bar{\Delta} K_{3p} \end{pmatrix} = \begin{pmatrix} -3.68 \\ -4.18 \\ -4.24 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -7 \\ -1.31 \\ -1.36 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -7 \\ -1.31 \\ -1.36 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} = \begin{pmatrix} 8.3 \\ -1.44 \\ -1.4 \end{pmatrix}. \tag{29}$$

5 CONCLUDING REMARKS

This study made an original contribution to the literature of economic growth by developing a nonlinear dynamic trade model with national government debts and national debts. In order to address issues related to dynamic relationships between global growth, trade, economic structural change, government's debts, and national debts, this study applies Zhang's approach to behavior of household to an extended Oniki-Uzawa model with governments' debts and endogenous public good supplies. The general dynamic equilibrium growth model is developed with any number of national economies and free trade between countries. The model is an integration of a few well-known theoretical models, the Solow growth model, the Uzawa two-sector growth model, the Oniki–Uzawa trade model, and Diamond's growth model with government's debt. A national economy is composed one tradable, one non-tradable and one public sectors. The dynamic interdependence between wealth accumulation, division of labor, governments' debts, national debts, and wealth and capital distribution for J-country world economy is described by $2J$ differential equations. We examined dynamic properties of the model by simulation. We found an equilibrium point and plotted the motion of the dynamic system. As the equilibrium point is locally unstable, we conducted short-run comparative dynamic analysis and comparative static analysis with regard to some parameters. It should be remarked that the economic structures and interactions in our model are delicately interrelated. Our comparative dynamic analysis is limited to a few cases. We might get more insights from further simulation. We may extend the model in some directions. The Solow model, the Uzawa two-sector growth, the Oniki-Uzawa trade model, and the Diamond model are most well-known models in the literature of growth theory. Many limitations of our model and possible extensions and generalizations become apparent in the light of the sophistication of the literature.

Appendix: Proving the lemma

By (3), (4) and (8) we obtain

$$z_j \equiv \frac{r + \delta_j}{w_j} = \frac{N_{jq}}{\bar{\beta}_{jq} K_{jq}}, \tag{A1}$$

where

$$\bar{\beta}_{jq} \equiv \frac{\beta_{jq}}{\alpha_{jq}}, \quad q = i, s, p.$$

From (A1) and (3), we obtain

$$r(z_1) = \alpha_j z_j^{\beta_{jq}} - \delta_j, \tag{A2}$$

where

$$\alpha_j \equiv \alpha_{ji} \bar{\tau}_j A_{ji} \bar{\beta}_{ji}^{\beta_{ji}}.$$

From (A2) we also have

$$z_j(z_1) = \left(\frac{r + \delta_j}{\alpha_j} \right)^{1/\beta_{ji}}. \tag{A3}$$

From (A1) we have

$$w_j(z_1) = \frac{r + \delta_j}{z_j}. \qquad \text{(A4)}$$

From (4) we have

$$p_j(z_1) = \frac{\bar{\beta}_{js}^{\alpha_{js}} w_j z_j^{\alpha_{js}}}{\bar{\tau}_{js} \beta_{js} A_{js}}. \qquad \text{(A5)}$$

From (6)-(8)

$$\hat{y}_j = \bar{\tau}_{jw} h_j w_j + (1 + \bar{\tau}_{ja} r) a_j. \qquad \text{(A6)}$$

From $p_j c_{js} = \gamma_j \hat{y}_j$ and (16)

$$N_{js} = \frac{\gamma_j \hat{y}_j \bar{N}_j}{p_j f_{js}}, \qquad \text{(A7)}$$

where

$$f_{js} \equiv \frac{F_{js}}{N_{js}} = \frac{A_{jq}}{(\bar{\beta}_{jq} z_j)^{\alpha_{jq}}}.$$

Insert (A6) in (A7)

$$N_{js} = n_{0j} + n_j a_j, \qquad \text{(A8)}$$

where

$$n_j(z_1) \equiv \frac{(1+\bar{\tau}_{ja} r)\gamma_j \bar{N}_j}{p_j f_{js}}, \quad n_{0j}(z_1) \equiv \frac{h_j \bar{\tau}_{jw} \gamma_j \bar{N}_j w_j}{p_j f_{js}}.$$

From (6) we have

$$Y_{jp} = \tau_j (F_{ji} + p_j F_{sj}). \tag{A9}$$

Insert (3) and (4) in (A9)

$$Y_{jp} = \tau_j w_j \left(\frac{N_{ji}}{\beta_{ji} \bar{\tau}_{ji}} + \frac{N_{js}}{\beta_{js} \bar{\tau}_{js}} \right). \tag{A10}$$

From (8) and (A10) we have

$$N_{jp} = \tau_j \beta_{jp} \left(\frac{N_{ji}}{\beta_{ji} \bar{\tau}_{ji}} + \frac{N_{js}}{\beta_{js} \bar{\tau}_{js}} \right). \tag{A11}$$

Insert (A11) in (18)

$$\left(1 + \frac{\tau_j \beta_{jp}}{\beta_{ji} \bar{\tau}_{ji}}\right) N_{ji} + \left(1 + \frac{\tau_j \beta_{jp}}{\beta_{js} \bar{\tau}_{js}}\right) N_{js} = N_j. \tag{A12}$$

Insert (A8) in (A12)

$$N_{ji} = \bar{n}_{0j} - \bar{n}_j a_j, \tag{A13}$$

where

$$\bar{n}_{0j} \equiv \left(N_j - \left(1 + \frac{\tau_j \beta_{jp}}{\beta_{js} \bar{\tau}_{js}}\right) n_{0j} \right) \left(1 + \frac{\tau_j \beta_{jp}}{\beta_{ji} \bar{\tau}_{ji}}\right)^{-1}, \quad \bar{n}_j \equiv \left(1 + \frac{\tau_j \beta_{jp}}{\beta_{js} \bar{\tau}_{js}}\right) \left(1 + \frac{\tau_j \beta_{jp}}{\beta_{ji} \bar{\tau}_{ji}}\right)^{-1} n_j.$$

Insert (A13) and (A8) in (A11)

$$N_{jp} = \tilde{n}_{0j} + \tilde{n}_j a_j, \tag{A14}$$

where

$$\tilde{n}_{0j} \equiv \left(\frac{\bar{n}_{0j}}{\beta_{ji} \bar{\tau}_{ji}} + \frac{n_{0j}}{\beta_{js} \bar{\tau}_{js}} \right) \tau_j \beta_{jp}, \quad \tilde{n}_j \equiv \left(\frac{n_j}{\beta_{js} \bar{\tau}_{js}} - \frac{\bar{n}_j}{\beta_{ji} \bar{\tau}_{ji}} \right) \tau_j \beta_{jp}.$$

From (A1), (A14), (A13) and (A8), we have

$$K_{jp} = \frac{\tilde{n}_{0j} + \tilde{n}_j a_j}{\bar{\beta}_{jp} z_j}, \quad K_{ji} = \frac{\bar{n}_{0j} - \bar{n}_j a_j}{\bar{\beta}_{ji} z_j}, \quad K_{js} = \frac{n_{0j} + n_j a_j}{\bar{\beta}_{js} z_j}. \tag{A15}$$

From (17) and (A15) we have

$$K_j = \hat{n}_{0j} + \hat{n}_j a_j, \tag{A16}$$

where

$$\hat{n}_{0j} \equiv \left(\frac{\tilde{n}_{0j}}{\bar{\beta}_{jp}} + \frac{\bar{n}_{0j}}{\bar{\beta}_{ji}} + \frac{n_{0j}}{\bar{\beta}_{js}} \right) \frac{1}{z_j}, \quad \hat{n}_j \equiv \left(\frac{\tilde{n}_j}{\bar{\beta}_{jp}} - \frac{\bar{n}_j}{\bar{\beta}_{ji}} + \frac{n_j}{\bar{\beta}_{js}} \right) \frac{1}{z_j}.$$

From (A16) and (19) we have

$$K = b_0 + \sum_{j=1}^{J} \hat{n}_j a_j, \tag{A17}$$

where

$$b_0 \equiv \sum_{j=1}^{J} \hat{n}_{0j}.$$

From (20) and (A17) we solve

$$a_1 = \Lambda\left(z_1, \{a_j\}, (D_j)\right) \equiv \frac{1}{\bar{N}_1 - \hat{n}_1}\left(b_0 + \sum_{j=2}^{J}\left(\hat{n}_j - \bar{N}_j\right)a_j + \sum_{j=1}^{N} D_j\right), \tag{A18}$$

where $\{a_j\} = (a_2, ..., a_J)$.

It is straightforward to check that all the variables can be expressed as functions of z_1, $\{a_j\}$ and (D_j) at any point in time as follows: r by (A2) $\rightarrow$ z_j by (A3) $\rightarrow$ w_j by (A4) $\rightarrow$ p_j by (A5) $\rightarrow$ a_1 by (A18) $\rightarrow$ N_{js} by (A8) $\rightarrow$ N_{ji} by (A13) $\rightarrow$ N_{jp} by (A14) $\rightarrow$ K_{jp}, K_{ji}, and K_{js} by (A1) $\rightarrow$ K_j by (A16) $\rightarrow$ K by (A17) $\rightarrow$ $\hat{y}_j$ by (A6) $\rightarrow$ F_{jq} by (1) $\rightarrow$ c_j, c_{js}, and s_j by (10) $\rightarrow$ Y_{jq} by (A10) $\rightarrow$ T_{jq} by (14). From this procedure and (13) and (15), we have

$$\dot{a}_1 = \Lambda_0\left(z_1, \{a_j\}, (D_j)\right) \equiv s_1 - a_1, \tag{A19}$$

$$\dot{a}_j = \Lambda_j\left(z_1, \{a_j\}, (D_j)\right) \equiv s_j - a_j, \quad j = 2, ..., J,$$
$$\dot{D}_j = \Psi_j\left(z_1, \{a_j\}, (D_j)\right) \equiv r D_j + Y_{jp} - T_{jp}, \quad j = 1, ..., J. \tag{A20}$$

Taking derivatives of (A18) with respect to t yields

$$\dot{a}_1 = \frac{\partial \Lambda}{\partial z_1}\dot{z}_1 + \sum_{j=2}^{J} \Lambda_j \frac{\partial \Lambda}{\partial a_j} + \sum_{j=1}^{J} \Psi_j \frac{\partial \Lambda}{\partial D_j}, \tag{A21}$$

where we also use (A20). From (A19) and (A21), we have

$$\dot{z}_1 = \Lambda_1\left(z_1, \{a_j\}, (D_j)\right) \equiv \left(\Lambda_0 - \sum_{j=2}^{J} \Lambda_j \frac{\partial \Lambda}{\partial a_j} - \sum_{j=1}^{J} \Psi_j \frac{\partial \Lambda}{\partial D_j}\right)\left(\frac{\partial \Lambda}{\partial z_1}\right)^{-1}. \qquad (A22)$$

We determine the motion of the system with (A20) and (A22) and the rest variables by the procedure provided before. In summary, we proved the lemma.

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