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A METHOD FOR RANKING DECISIONAL ALTERNATIVES USING INTUITIONISTIC FUZZY SETS

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Abstract. *This paper proposes a new method for ranking the alternatives represented under the form of IFS. The method is analysed in comparison with other techniques used for ordering introduced by Xu (2007), Szmidt and Kacprzyk (2008). The order is determined based on the distance to the ideal solution (the optimal point) and on the distance to the points* P_{pes} and P_{ned} *corresponding to the maximum level of non-membership, and of indeterminacy, respectively. The proposed method is applied in a multi-attribute decision model.*

Keywords: ranking alternatives, intuitionistic fuzzy sets, multi-attribute decision models.

1. INTRODUCTION

Assisting business decision-making through systematic analysis methods and models for decision-making is required given the limits of the human decision-maker. Filip (2002) presents the main types of generally applicable limits:

- cognitive limitations related to the manager's limited ability to store and process information and knowledge;
- economic limitations related to the cost of obtaining and processing information;
- time limits reflected in the poor quality and incorrect decisions taken under time pressure in a competitive environment.

These limits are generally amplified by the characteristics of the economic environment.

Managers have to cope with complex and contradictory multi-criteria decision problems, for example: obtaining bigger profits, minimizing costs and risks. Such issues arise in: investment processes, human resource selection, choosing the marketplaces. Another important class of multi-criteria decision problems occurs in the case of policy decisions (Filip, 2002).

Generally, these decisional situations lead to problems with a limited number of discrete alternatives and, consequently, the fuzzy techniques can be used in the construction of the model.

The concept of fuzzy set (fuzzy set, ensemble flou) was introduced in mathematics in 1965, by L.A. Zadeh, a professor at the University of California at Berkeley. Fuzzy concepts arose from the need to measure the vague, the imprecise quantitatively. Zadeh mathematically underlies the idea of incomplete membership to a set and he introduces the degree of membership of elements (represented by a real number between 0 and 1) which defines the fuzzy set.

Zadeh (1965) asserts that the intention of the fuzzy sets theory is not to replace the probability measures of random stochastic processes, but to offer "... a natural approach to the problems in which the source of imprecision is given by the absence of clearly defined criteria of membership to a class, and not by the presence of a random variable".

Kaufmann (1998) describes the theory of fuzzy sets as *"... a system of concepts and techniques that gave a norm of mathematical precision to human cognitive processes, which in many cases are vague and ambiguous in relation to the standards of classical mathematics"*.

Zimmermann (1985) states: *"The theory of fuzzy sets has a strict mathematical framework, in which vague conceptual phenomena can be precisely and rigorously studied"*.

The Romanian academician Grigore Moisil (1975) developed the idea of fuzzy set, which could be understood as the extension of a predicate in logics, with an infinite number of values.

2. PRELIMINARIES

In (Zadeh, 1965) a fuzzy set is defined as follows: let $X=\{x_1,x_2,...,x_n\}$ be a classical set; the fuzzy set *A* is characterized by the *membership function* $\mu_A : X \to [0,1]$, that associates the degree of membership $\mu_A(x_j)$ to each element $x_j \in X$,

$$A=\{(x_j,\mu_A(x_j)),x_j \in X\} \quad (1)$$

In the specific case when μ_A only takes the values 0 or 1, the fuzzy set *A* is identical to a classical subset of X.

Let $e_i=(x_i,\mu_A(x_i))$ and $e_j=(x_j,\mu_A(x_j))$ be two elements of the fuzzy set *A*.

We say that:

$$e_i < e_j \quad if \quad \mu_A(x_i) < \mu_A(x_j) \quad (2)$$

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The set *IFS* (*Intuitionistic Fuzzy Set*) *A* in *X*, noted in what follows as *IFS A*, was introduced by Atanasov (1999), as a generalization of the fuzzy set. We define:

$$A=\{(x_j,\mu_A(x_j),\nu_A(x_j)),x_j\in X\} \quad (3)$$

IFS A is characterized by the membership function μ_A and by the non-membership function ν_A, where:

$$\mu_A : X\rightarrow[0,1], \quad x_j\in X\rightarrow\mu_A(x_j)\in[0,1] \quad (4)$$

$$\nu_A : X\rightarrow[0,1], \quad x_j\in X\rightarrow\nu_A(x_j)\in[0,1] \quad (5)$$

on the condition that: $\mu_A(x_j)+\nu_A(x_j)\le 1, \quad \forall x_j\in X$

For any set *IFS A*, we define the degree of indeterminacy (*intuitionist fuzzy index, hesitation margin*) of x_j, by:

$$\pi_A(x_j)=1-\mu_A(x_j)-\nu_A(x_j) \quad (6)$$

This one expresses a lack of knowledge as regards the membership or non-membership of *x* to *A*.

It is obvious that

$$0\le\pi_A(x)\le 1, \quad \forall x\in X$$

If $\pi_A(x_j)=1-\mu_A(x_j)-\nu_A(x_j)=0, \quad \forall x_j\in X$, then *IFS A* is reduced to a classical fuzzy set (Xu, 2007a).

It is important to consider *the degree of indeterminacy* $\pi_A(x)$ in all measures defined for distance, entropy, similarity (D. Guha, D. Chakraborty, 2010), (Szmidt E., J. Baldwin, 2004) (Szmidt E., J. Baldwin, 2005) (Szmidt E., J. Kacprzyk, 2005a) (Szmidt E., J. Kacprzyk, 2005b), (Xu, Z., 2007a) (WL Hung, Yang MS, 2007), (WL Hung Yang MS, 2008), (Szmidt E., J. Kacprzyk, Bujnowski P., 2014).

3. GEOMETRIC INTERPRETATION

As for any element $x\in X$ $\mu_A(x)+\nu_A(x)+\pi_A(x_j)=1$ is valid and each of the terms of the sum belongs to the interval $[0,1]$, we can imagine the unit cube (Figure 1), in which the triangle *ABC* is the surface of the points which satisfy the above equation. Each point is described by three coordinates (μ,ν,π). Thus, point $A(1,0,0)$ represents the element that completely belongs to *IFS*, as $\mu=1$. Point $B(0,1,0)$ represents the element which does not belong to *IFS*,

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as $\nu = 1$. Point $C(0,0,1)$ is the element we can say nothing about as regards the membership or non-membership to *IFS*, as $\pi = 1$. The segment *AB*, characterized by $\pi = 0$, represents the elements that belong to a classical fuzzy set $(\mu + \nu = 1)$. In conclusion, every element that belongs to an *IFS* can be mapped to a point that belongs to the triangle *ABC*.

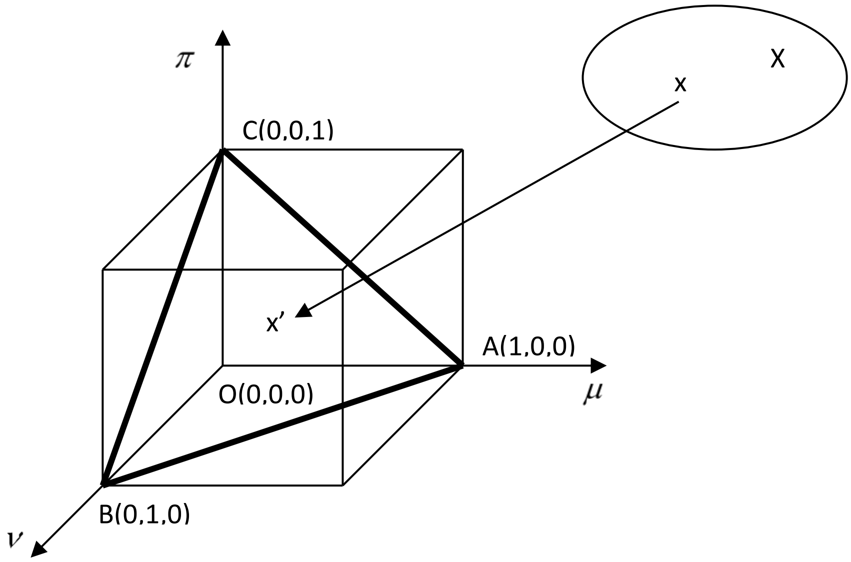

Fig. 1. The 3D geometric representation of an IFS set

(Szmidt E., J. Kacprzyk, Bujnowski P., 2014)

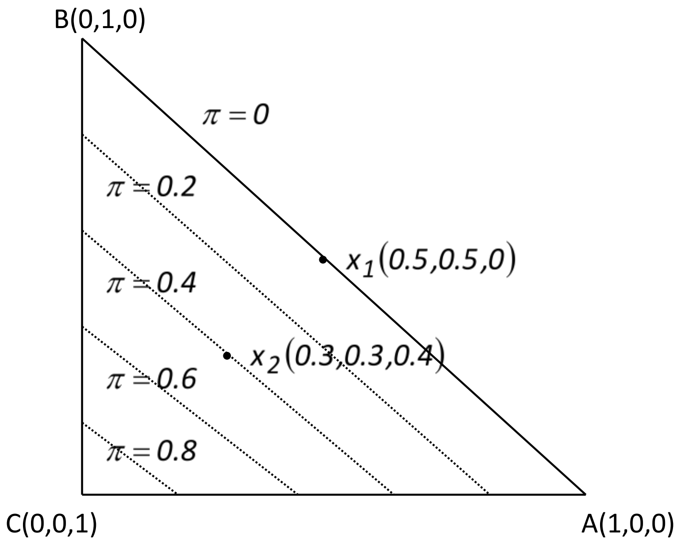

Fig. 2. The 2D geometric representation of an IFS set

(Szmidt E., J. Kacprzyk, Bujnowski P., 2014)

4. METHODS FOR SORTING *IFS* ELEMENTS

4.1. A method based on the associated fuzzy number

In (Xu, 2007b) an *IFN* (*intuitionistic fuzzy number*) $\bar{x}_j = (\mu_A(x_j), \nu_A(x_j))$ is associated to each element of the IFS, $A = \{(x_j, \mu_A(x_j), \nu_A(x_j)), x_j \in X\}$,

We define *the score* of $\bar{x}_j$:

$$s(\bar{x}_j) = \mu_A(x_j) - \nu_A(x_j), \quad where \quad s(\bar{x}_j) \in [-1,1] \tag{7}$$

and the degree of accuracy of the IFN $\bar{x}_j$:

$$h(\bar{x}_j) = \mu_A(x_j) + \nu_A(x_j), \quad where \quad h(\bar{x}_j) \in [0,1] \tag{8}$$

On the *IFN* set, an order relationship can be defined (Xu, 2007b), as follows:

$$\bar{x}_i < \bar{x}_j \quad if \quad s(\bar{x}_i) < s(\bar{x}_j) \quad or$$

$$if \quad s(\bar{x}_i) = s(\bar{x}_j) \quad and \quad h(\bar{x}_i) < h(\bar{x}_j) \tag{9}$$

$$\bar{x}_i = \bar{x}_j \quad if \quad s(\bar{x}_i) = s(\bar{x}_j) \quad and \quad h(\bar{x}_i) = h(\bar{x}_j) \tag{10}$$

The equality $\bar{x}_i = \bar{x}_j$ implies: $\mu_A(x_i) = \mu_A(x_j)$ and $\nu_A(x_i) = \nu_A(x_j)$.

The relationship defined allows a classification of the elements of IFS set A to be established.

4.2. A distance-based method

In (Szmidt E., J. Kacprzyk, 2008) a classification method is presented based on the distance to the element with the highest degree of membership, but which also considers the reliability of information, i.e. how safe it is. The reliability of information is given by *the degree of indeterminacy* $\pi_A(x_j)$. A high *degree of indeterminacy* shows a lack of knowledge as regards the membership or non-membership of *x* to *IFS A*. This indicates a low reliability of the information relative to the element x_j.

To calculate the distance between two sets *IFS A* and *B* in $X = \{x_1, x_2, ..., x_n\}$, the normalized Hamming *distance* formula is used:

$$d_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right) \tag{11}$$

We have $0 \le d_{IFS}(A,B) \le 1$. E Szmidt E. and Kacprzyk J. propose that a ranking index should be calculated for each element of the set *IFS A*.

$$R(x_i) = 0.5 \cdot (1 + \pi_A(x_i)) \cdot d_{IFS}(P_{opt}, x_i) \tag{12}$$

where: $P_{opt} \in X$ and $(P_{opt}, 1, 0)$, meaning that $\mu_A(P_{opt}) = 1$ and $\nu_A(P_{opt}) = 0$

and $$d_{IFS}(P_{opt}, x_i) = \frac{1}{2}\left(|1 - \mu_A(x_i)| + |0 - \nu_A(x_i)| + |0 - \pi_A(x_i)|\right) \tag{13}$$

It should be noted that P_{opt} corresponds to the point $A(1,0,0)$ from Figure 2.

The classification is obtained by sorting the values of the rank indexes $R(x_i)$ in ascending order.

4.3. A new distance-based method for sorting IFS elements

The method that we propose considers the distances to three elements:

the element with *the highest degree of membership* P_{opt}, $P_{opt} \in X$ and $(P_{opt}, 1, 0)$, that is $\mu_A(P_{opt}) = 1$, $\nu_A(P_{opt}) = 0$ and $\pi_A(P_{opt}) = 0$; it corresponds to the point $A(1,0,0)$ in Figure 2;

the element *with the highest degree of non-membership*; we note it with P_{pes}, $P_{pes} \in X$ and $(P_{pes}, 0, 1)$, that is $\mu_A(P_{pes}) = 0$, $\nu_A(P_{pes}) = 1$ and $\pi_A(P_{pes}) = 0$; it corresponds to the point $B(0,1,0)$ in Figure 2;

the element *with the highest degree of indeterminacy*; we note it with P_{ned}, $P_{ned} \in X$ and $(P_{ned}, 0, 0)$, that is $\mu_A(P_{ned}) = 0$, $\nu_A(P_{ned}) = 0$ and $\pi_A(P_{ned}) = 1$; it corresponds to the point $C(0,0,1)$ in Figure 2;

For each element of the set *IFS A* we calculate:

$$d_1 = d_{IFS}(P_{opt}, x_i) = \frac{1}{2}\left(|1 - \mu_A(x_i)| + |0 - \nu_A(x_i)| + |0 - \pi_A(x_i)|\right) \tag{14}$$

$$d_2 = d_{IFS}(P_{pes}, x_i) = \frac{1}{2}\left(|0 - \mu_A(x_i)| + |1 - \nu_A(x_i)| + |0 - \pi_A(x_i)|\right) \tag{15}$$

$$d_3 = d_{IFS}(P_{ned}, x_i) = \frac{1}{2}\left(|0 - \mu_A(x_i)| + |0 - \nu_A(x_i)| + |1 - \pi_A(x_i)|\right) \tag{16}$$

The better an item will be classified, the smaller the distance d_1 is, and the bigger the distances d_2 and d_3 are. Nevertheless, we will penalize more the distance d_2 to P_{pes} as compared to the distance d_3 to P_{ned}, given that non-membership is more important than indeterminacy.

For this, we determine a ranking index, $Q(x_i)$, for each element $x_i \in X$ of the set *IFS A*, depending on the distances to the elements $P_{opt}, P_{pes}, P_{ned}$. We propose as a formula for the determination of *the rank index*:

$$Q(x_i) = \frac{3 \cdot d_1}{d_1 + 3 \cdot d_2 + 2 \cdot d_3} \tag{17}$$

The classification is obtained by sorting the values of the rank index $Q(x_i)$ in ascending order.

Remark.

The relationships (14), (15) and (16) which express the distance to the points P_{opt}, P_{pes} and P_{ned} become:

$$d_1 = d_{IFS}(P_{opt}, x_i) = \frac{1}{2}\left(\left|1 - \mu_A(x_i)\right| + \left|0 - \nu_A(x_i)\right| + \left|0 - \pi_A(x_i)\right|\right) =$$

$$= \frac{1}{2}\left(1 - \mu_A(x_i) + \nu_A(x_i) + \pi_A(x_i)\right) = 1 - \mu_A(x_i)$$

because, according to equation (6), we have: $\mu_A(x_i) + \nu_A(x_i) + \pi_A(x_i) = 1$.

Analogously, $d_2 = d_{IFS}(P_{pes}, x_i) = 1 - \nu_A(x_i)$ and $d_3 = d_{IFS}(P_{ned}, x_i) = 1 - \pi_A(x_i)$.

It follows that equation (18) becomes:

$$Q(x_i) = \frac{3 \cdot (1 - \mu)}{(1 - \mu) + 3 \cdot (1 - \nu) + 2 \cdot (1 - \pi)} = \frac{3 \cdot (1 - \mu)}{5 - (2 \cdot \nu + \pi)} \tag{18}$$

Examples

The method based on the associated fuzzy number

IFS Element		Score	Degree of accuracy	Rank
P_{opt}	$(1,0,0)$	1	1	1
P_{pes}	$(0,1,0)$	-1	1	9
P_{ned}	$(0,0,1)$	0	0	7
X_1	$(0.4,0.4,0.2)$	0	0.8	6

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x_2	$(0.5,0.3,0.2)$	0.2	0.8	3
x_3	$(0.5,0.2,0.3)$	0.3	0.7	2
x_4	$(0.5,0.5,0)$	0	1	5
x_5	$(0.2,0.1,0.7)$	0.1	0.3	4
x_6	$(0.2,0.7,0.1)$	-0.5	0.9	8

Distance- based methods

IFS Element		Rank index R(x)	Rank	Rank index Q(x)	Rank
P_{opt}	$(1,0,0)$	0.00	1	0.00	1
P_{pes}	$(0,1,0)$	0.50	7	1.00	9
P_{ned}	$(0,0,1)$	1.00	9	0.75	8
x_1	$(0.4,0.4,0.2)$	0.36	5	0.45	5
x_2	$(0.5,0.3,0.2)$	0.30	3	0.36	3
x_3	$(0.5,0.2,0.3)$	0.33	4	0.35	2
x_4	$(0.5,0.5,0)$	0.25	2	0.38	4
x_5	$(0.2,0.1,0.7)$	0.68	8	0.59	6
x_6	$(0.2,0.7,0.1)$	0.44	6	0.69	7

5. A MODEL OF MULTIATTRIBUTE DECISION-MAKING

We consider a classic problem of multi-attribute decision-making. We have:

$A = \{A_1, A_2, ..., A_m\}$ - the set of alternatives

$C = \{C_1, C_2, ..., C_n\}$ - the set of characteristics (attributes)

$W = \{w_1, w_2, ..., w_m\}$ - the weights associated to the characteristics, where

$w_i \geq 0, \quad i = 1,2,...,m$ şi $\sum_{i=1}^{m} w_i = 1$

$V_{ij}, i = 1,2,...,m, \quad j = 1,2,...,n$ - the values of alternatives for each feature.

The diameters method is a direct method for ranking of alternatives. The method considers the degree of homogeneity of appreciations as compared to the attributes. In order to avoid compensation, two functions are defined: *an appreciation function* and *a diameter function*. Thus, the more homogenous a variant is, the smaller its diameter is, and the better it is, the greater the appreciation is.

The algorithm of the method is as follows:

Step 1. We calculate *the appreciation function*: $\varphi : A \rightarrow R$, defined as follows

$$\varphi(A_i) = \sum_{j=1}^{n} (m - pos(A_i, C_j)) \cdot w_j, \; i = 1,2,...,m \tag{19}$$

where: $pos : A \times C \rightarrow \{1,2,...m\}$ and $pos(A_i, C_j) = k$, i.e. the value of the alternative A_i for the attribute *j* takes the place *k* in the (ascending / descending) hierarchy of the *m* values associated to the attribute *j*, as this is the minimum / maximum.

Step 2. *The diameter function* is defined: $\delta : A \rightarrow N$

$$\delta(A_i) = \max_j(pos(A_i, C_j)) - \min_j(pos(A_i, C_j)) \quad i = 1,2,...,m \quad j = 1,2,...,n \tag{20}$$

Step 3. *The aggregate function* is calculated: $\varphi \& \delta : A \rightarrow R$

$$\varphi \& \delta(A_i) = \frac{(\varphi(A_i) + (m - \delta(A_i)))}{2}, \quad i = 1,2,...,m \tag{21}$$

Step 4. The hierarchy of the alternatives is given by the descending values of the function $\varphi \& \delta$. *STOP*.

The adaptation of the diameters method when the values of the alternatives for each characteristic are represented as *IFS* leads to the following algorithm for calculating the aggregate function $\varphi \& \delta(A_i), i = 1,2,...,m$, which will enable the decision alternatives to be sorted (Lixandroiu D., Lixandroiu R., 2013) (Lixandroiu D., 2011).

Step 1. Input m - number of alternatives

n – the number of characteristics (attributes) for each alternative

$W=\{w_1,w_2,...,w_n\}$ - attributes weights

$A_i=\{(C_j,\mu_{A_i}(C_j),\nu_{A_i}(C_j)),C_j\in C\},\quad i=1,2,...,m$

where: $\mu_{A_i}(C_j)$ indicates the degree to which the alternative A_i satisfies the attribute C_j and $\nu_{A_i}(C_j)$ indicates the degree to which the alternative A_i does not satisfies the attribute

Step 2. For each attribute, we determine the rank of the values by applying one of the methods already presented. We obtain the matrix:

$$P=(pos(A_i,C_j),i=1,2,...,m,j=1,2,...,n)$$

Step 3. We calculate the values of the appreciation function $\varphi(A_i),i=1,2,...,m$, according to (19).

Step 4. We calculate the diameter function $\delta(A_i),i=1,2,...,m$ according to (20).

Step 5. We calculate the aggregate function $\varphi\&\delta(A_i),i=1,2,...,m$, according to (21) and we determine the order of the decisional alternatives. *STOP*.

Numerical example.

We apply *the diameters method* to rank five decision alternatives, according to the values of four characteristics (attributes). These values are expressed as *IFS*. To determine the rank of the values we apply the new distance based method proposed.

The problem. (Bojadziev, G., Bojadziev, M., 1995). We suppose that the set of alternatives *A* consists of five candidates ($A_i,i=1,2,...,5$) who apply for a position in an organization. The selection committee wants to choose the candidate who satisfies a set of three objectives to the highest degree: O_1, O_2, O_3. The selection committee has a restriction *R* as regards the wage given and accepted should be as small as possible. After analysing the CVs, the letters of recommendation and following the interviews, each member of the commission sets for each candidate the degree of membership to the set of objectives and to the restriction discussed during the interview in connection to the accepted salary. The sets defined as *IFS* resulted:

$$A_i = \{(C_j, \mu_{A_i}(C_j), \nu_{A_i}(C_j)), C_j \in C\},\ i = 1,2,...,m.$$

The application of the algorithm leads to:

Step 1. Entry $m = 5, n = 4$

A_1 = {(C_1, 0.8, 0.1), (C_2, 0.7, 0.1), (C_3, 0.7, 0), (C_4, 0.4, 0.3)}

A_2 = {(C_1, 0.6, 0.3), (C_2, 0.6, 0.1), (C_3, 0.8, 0.1), (C_4, 0.7, 0.2)}

A_3 = {(C_1, 0.3, 0.5), (C_2, 0.8, 0.1), (C_3, 0.5, 0.3), (C_4, 0.6, 0.3)}

A_4 = {(C_1, 0.7, 0.1), (C_2, 0.2, 0.5), (C_3, 0.5, 0.3), (C_4, 0.8, 0.1)}

A_5 = {(C_1, 0.5, 0.2), (C_2, 0.3, 0.6), (C_3, 0.4, 0.2), (C_4, 0.9, 0)}

We suppose that the importance of the criteria is given by the vector

$W = \{0.2, 0.3, 0.2, 0.3\}$.

Step 2. For each characteristic we reorder the elements using the proposed method of 3 distances and we calculate the ranking index Q (x). The obtained matrix *P* is:

	C1		***C2***		***C3***		***C4***	
A1	*0.13*	***1***	*0.20*	***2***	*0.19*	***2***	*0.44*	***5***
A2	*0.28*	***3***	*0.27*	***3***	*0.13*	***1***	*0.20*	***3***
A3	*0.55*	***5***	*0.13*	***1***	*0.36*	***3***	*0.28*	***4***
A4	*0.20*	***2***	*0.65*	***5***	*0.36*	***3***	*0.13*	***2***
A5	*0.35*	***4***	*0.57*	***4***	*0.43*	***4***	*0.06*	***1***

Step 3. We calculate: $\varphi(A_i),\ i = 1,2,...,m$

$$\varphi(A_1) = (5-1)\cdot 0.2 + (5-2)\cdot 0.3 + (5-2)\cdot 0.2 + (5-5)\cdot 0.3 = 2.3$$

Analogously $\varphi(A_2) = 2.4$ $\varphi(A_3) = 1.9$ $\varphi(A_4) = 1.9$ $\varphi(A_5) = 1.9$

Step 4. We calculate $\delta(A_i), i = 1,2,...,m$

It results: $\delta(A_1) = 5 - 1 = 4$ $\delta(A_2) = 2$ $\delta(A_3) = 4$ $\delta(A_4) = 3$

$\delta(A_5) = 3$

Step 4. We calculate $\varphi \& \delta(A_i), i = 1,2,...,m$

It results: $\varphi\&\delta(A_1)=[2.3+(5-4)]/2=1.65$

$\varphi\&\delta(A_2)=2.70 \quad \varphi\&\delta(A_3)=1.45 \quad \varphi\&\delta(A_4)=1.95 \quad \varphi\&\delta(A_5)=1.95$

The order of the alternatives is: $A_2 \succ A_4 = A_5 \succ A_1 \succ A_3$ and, consequently, we select candidate number 2. *STOP*.

6. CONCLUSIONS

To calculate the ranking index *Q(x)*, the proposed method for ranking the *IFS* elements considers the distances to three elements IFS, corresponding to the points $A(1,0,0), B(0,1,0)$, $C(0,0,1)$ from the 3D representation shown in Figure 1. Non-membership (ν) is penalized more than indeterminacy (π), because it is considered more important than indeterminacy, which can be interpreted as a temporary lack of information, which can evolve into membership or non-membership.

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