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ESTIMATION OF COB-DOUGLAS AND TRANSLOG PRODUCTION FUNCTIONS WITH CAPITAL AND GENDER DISAGGREGATED LABOR INPUTS IN THE USA

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ABSTRACT: *This is an empirical investigation of the homogeneity of gender disaggregated labor using the Douglas, single/multi-factor translog production functions; and labor productivity functions for the USA. The results based on the single factor translog model, indicated that: an increase in the capita/female labor ratio increases aggregate output; male labor is more productive than female labor, which is more productive than capital; a simultaneous increase in quantity allocated and productivity of the leads to an increase in output; female labor productivity has grown slower than male labor productivity; it much easier to substitute male labor for capital compared to female labor; and the three inputs are neither perfect substitutes nor perfect complements. As a consequence, male and female labor are not homogenous inputs. Efforts to investigate the factors influencing gender disaggregated labor productivity; and designing policies to achieve gender parity in numbers/productivity in the labor force and increasing the ease of substitutability between male labor and female labor are required.*

JEL CLASSIFICATION: C13, C20, C22, C32, C51, C52, D24, E23, E24, J24, O47, O51, B54

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1. INTRODUCTION

Several studies such as, Pavelescu (2011), Parlinska and Dareev (2011), Chongela, Nandala, and Korabandi (2013), Helali and Kalai (2015), Napasintuwong and Emerson (2015), investigated the productivity of various inputs for in production processes. However, in these studies and others, the labor input has been treated as a homogenous variable, with no distinction between male and female labor used in the production process. In so doing, they have overlooked the discrimination between male and female labor which has existed and continues to exist in many production processes world over, with male labor being preferred over female labor in certain sectors and vice versa, leading to occupational segregation in the labor market and the associated differences in productivities as well as the persistent gender wage gap.

The purpose of this study is to fill this gap by using a Cobb-Douglas (CD) and a transcendental logarithmic or translog (TL) production functions with gender disaggregated labor, implying non-homogenous male and female labor inputs; and draw empirical implications using the USA data. This will specifically involve: estimating the CD and TL (both single and multiple input) functions) and the using capital, and either gender disaggregated or aggregate labor inputs; performing tests to determine whether the functions are appropriate; estimating the output elasticities (elasticities of scale), average elasticities of scale, marginal products (measure productivity of capital and labor inputs (aggregate, male and female), the marginal rates of substitution between labor (both aggregate and gender disaggregated labor) and capital on one hand, and between female and male labor on the other hand, as well as the corresponding elasticities of substitution; and determining the appropriate production function with gender disaggregated labor for the USA. Ultimately, the study will determine whether male and female labor are homogenous in the production process, and whether they are perfect substitutes or complements in the production process.

2. METHODOLOGICAL LITERATURE REVIEW

Several researchers have used the TL (Collard-Wexler, 2012, Helali and Kalai, 2015, Khalil, 2005, Krishnapillai and Thompson, 2012, Njeru, 2010, Pavelescu, 2011) proposed by Christensen, Jorgenson and Lau (1971, 1973); the CD (Debertin, 2012, Parlinska and Dareev, 2011); and the Constant elasticity substitution (CES) (Chongela, Nandala, and Korabandi, 2013, Helali and Kalai, 2015, Juselius, 2008, Papageorgio and Saam, 2008) production functions.

The CD function has several restrictions including: assuming homogeneity (constant returns to scale) and unitary elasticity of substitution between input pairs (Debertin, 2012, Green, 2012), which may be very restrictive in terms of certain production activities. The TL function, on the other hand, relaxes the above assumptions by assuming flexible, stable over time, non –unitary elasticities of substitution (Allen and Hall, 1997, Green, 2012). Unlike the CD function, it does not assume perfect or smooth substitution between inputs or perfect competition on the factors market (Klacek, Vosvrda, and Schlosser al, 2007) and also caters for the transition from a linear relationship between the output and the inputs to a non- linear one.

The CES function unlike the CD function requires constant pair-wise elasticities of substitution which are equal for all inputs but unlike the CD function, it does not restrict it to a specific number but both functions are restrictive since they require invariant returns to scale and elasticities of substitution across input points). The elasticity of substitution for the CES function varies between 0 (Leontief production function) and infinity (linear production function), making the CD function (elasticity of substitution equal to 1-Confirm) a special case of the CES function.

The TL function can be used to: approximate the CES production function, especially when the elasticity of substitution is close to one and the second order approximation of linear-homogenous production; and to estimate the Allen elasticities of substitution, the production frontier or the measurement of total factor productivity dynamics. The usefulness of the TL production is limited by the fact that the number of parameters increase or explode as the number of inputs increases, and this leads to multi-collinearity (Boisvert, 1982) which end up having signs that are contrary to the expected sign of the coefficient of the correlation between the dependent variable and the analyzed explanatory (Pavelescu, 2010b). This can be overcome by limiting the number of factors of production to those that are ultimately important for the behaviour of the output and/or increasing the sample size (Boisvert, 1982 and Pavelescu, 2011). The importance of a given factor can be established by estimating the TL for a single factor and if useful, the respective input is introduced in the extended TL function, which is estimated using ordinary least squares (OLS) for only those variables that have been proved to have significance.

For empirical analysis, the choice of the appropriate function form depends on the theoretical consistency, domain applicability, flexibility, factual conformity and computation facility, however, no single function satisfies all the requirements, thus the best option should be identified and applied (See Lau, 1986).

3. METHODOLOGY

3.1 Two/three input Cobb-Douglas production functions

The CD function is first order Taylor expansion and is given by

$$Y = AK^{\alpha_k} L_T^{\alpha_l} \tag{1}$$

and in logarithmic form,

$$\ln Y = \ln A_{KL_1} + \alpha_K . \ln K + \alpha_L . \ln L_T \tag{2}$$

It assumes perfect substitution (the elasticity of substitution parameter is constant and is equal to one (implying a substitution parameter of 0); sometimes it is restricted to constant returns to scale whereby $\alpha_k + \alpha_L = 1$, although it can have either increasing returns ($\alpha_k + \alpha_L \succ 1$) or decreasing returns to scale($\alpha_k + \alpha_L \prec 1$). The three input CD function with gender disaggregated labor (male labor (L_m), female labor (L_m)) is given by

$$\ln Y = \ln A_{KL_{dg}} + \alpha_{K_{dg}}.\ln K + \alpha_{L_m}.\ln L_m + \alpha_{L_f} \ln L_f \tag{3}$$

with constant returns to scale represented by $\alpha_k + \alpha_{Lm} + \alpha_{L_f} = 1$.

3.2 Translog production function

3.2.1 Two input translog function: capital and aggregate labor

Given the production output measured in production values all expressed in logarithms, the second order Taylor expansion is represented by two possible functions represented in equation 4 (developed by Kmenta, 1967 which he used to approximate the CES production function) and equation 5, proposed by Christensen, Jorgenson and Lau, 1971, 1973). Equation 4 can be modified to yield the productivity function while equation 5 can be used to relax the homotheticity assumptions.

$$InY = InA_1 + \alpha_1.InK + \beta_1.InL_T + \chi_1.In^2(K / L_T) \tag{4}$$

Where

$\ln Y$ = logarithm of output measured by gross domestic product (GDP)

$\ln K$ = logarithm of capital employed measured by fixed capital

In = natural logarithms

Y = output

K = Fixed capital

$\ln L_T$ = logarithm of labor (total employed population or Total labor employment)

A1, α_1, β_1, and χ_1 are parameters to be estimated.

$$\ln Y = \ln A_{KL_1} + \alpha_K.\ln K + \alpha_L.\ln L_T + \beta_{K^2} \ln^2 K + \beta_{L^2} \ln^2 L_T + \beta_{KL}.\ln K.\ln L_T \tag{5}$$

Equation 5 can be rewritten in general function form for *n* inputs as

$$\ln Y = \ln A_{\alpha_i \beta_{ij}} + \sum_{i=1}^{n} \alpha_i . \ln X_i + (\frac{1}{2}) . \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} . \ln X_i . \ln X_j \qquad (6)$$

For n inputs, the number of estimated parameters given by $\frac{n.(n+3)}{2}$. The Cobb-Douglas production being the first order Taylor series can be obtained from equation 5 or 6 by setting $\beta_{ii} = \beta_{ij} = 0$, for $i \neq j$, implying that $\beta_{K^2} = \beta_{L^2} + \beta_{KL} = 0$

Imposing the condition of $\alpha_1 + \beta_1 = 1$, following Grilichs and Ringstad (1971), the TL function yields the labor productivity function in equation 7, which is expressed in form of a single input- represented by the capital-labor ratio.

$$In(Y / L_T) = InA_{2_1} + \alpha_2 . In(K / L_T) + \chi_1 . In^2 (K / L_T) \qquad (7)$$

To investigate the productivity of male (female) labor, equation 7 is estimated using male (female) labor instead of aggregate labor.

3.2.2 Three input translog production function with gender disaggregation labor input

The three input TL function with gender disaggregated labor (female and male labor employed) is obtained by expanding equation 6, thus

$$\ln Y = \ln A_{KL_{dg}} + \alpha_{K_{dg}} . \ln K + \alpha_{L_m} . \ln L_m + \alpha_{L_f} \ln L_f + \beta_{K_{dg}^2} \ln^2 K_{dg} + \beta_{L_m^2} \ln^2 L_m + \beta_{L_f^2} \ln^2 L_f$$

$$+ \beta_{KLm} . \ln K . \ln L_m + \beta_{KL_f} . \ln K . \ln L_f + \beta_{L_M L_F} . \ln L_M . \ln L_f \qquad (8)$$

In order to deal with the multi-collinearity that might arise from increase in the number of parameters, each factor will be tested using the test for significance of the factor and will only be included in the final translog equation if and only if it is significant. Multi-collinearity is undesirable since it may lead to wrong signs of the parameters, unstable parameter estimates thus, high variance. The procedure for testing the single factors is described in the next section. In the case of harmful collinearity for all variables, the multi-variable translog production cannot be used for any meaningful interpretation in terms of the precise effect of the predictors, and should not be estimated if this is the main goal of the analysis.

3.2.3 Main indicators derived from the translog production function parameters

The indicators which will be computed include: i) output elasticities for aggregate/gender disaggregated labor and capital; ii) the marginal products of the inputs capital and the labor (aggregate, female and male labor); ii) the marginal rate of transformation between: female labor and male labor, male labor and capital, female labor and capital, and between capital and total labor. The equations for computation of the above indicators are presented below.

a) Output Elasticities (Elasticity of Scale)

The output elasticity for each input in a multi input TL function is given by equation 9, a Cobb-Douglas function with $\frac{1}{2}\beta_{ij} = \varphi_{ij}$.

$$\alpha_{mi} = \frac{\delta \ln Y}{\delta \ln X_i} = \alpha_i + \frac{1}{2}\sum_{j=1}^{n}\beta_{ij} \ln X_j = \alpha_i + \sum_{j=1}^{n}\varphi_{ij} \ln X_j \tag{9}$$

where α_{mi} = the output elasticity for input X_i taking into account all the effects of the other factors in the translog function and $\frac{1}{2}\beta_{ij}$ is the coefficient of the multiplicative term, $\ln X_i . \ln X_j$ in equation 4 above or the value of the parameter (coefficient) of $\ln X_i . \ln X_j$ obtained in the empirical TL equation. Empirically, the output elasticity can estimated using equation 9.

b) Marginal Product

The marginal product (MP_{YX}) for the TL function is given by

$$MP_{YX_i} = \frac{\delta Y}{\delta X_i} \quad (10)$$

But

$$\alpha_{mi} = \frac{\delta \ln Y}{\delta \ln X_i} = \frac{\delta Y}{\delta X_i} \cdot \frac{X}{Y} \quad (11)$$

Rearranging equation 11, making the marginal product the subject, we obtain

$$MP_{YX_i} = \frac{\delta Y}{\delta X_i} = \frac{\delta \ln Y}{\delta \ln X_i} \cdot \frac{Y}{X} = \left[\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln X_j \right] \cdot \frac{Y}{X} \quad (12)$$

Thus, the marginal product is the product of the output elasticity and the ratio of the output to the input being investigated. It can be obtained for each set of Y and X in the data set, therefore a trend of the productivities can be established.

c) Marginal Rate of Substitution

For perfect substitutes, where both factors of production are identical for all purposes, the marginal rate of substitution is constant and uniform, implying that the elasticity of substitution between the two factors is infinite (with $\Delta MRTS_{K,L} = 0$ or $\frac{L}{K} = 0$) and the isoquants are straight lines. Production can be done using both inputs or only one of the inputs. It occurs using only the cheaper factor. A change in the relative price of factor will induce substitution to that factor that is cheaper relative to other after the change.

For perfect complements, where inputs have to be combined in fixed proportions to produce a certain amount of output ($\Delta(\frac{L}{K}) = 0$, the MRTS is infinite or zero (output does not increase by substitution of one input for the other, implying that the elasticity of substitution is zero and no substitution can occur. This implies L shaped isoquants (zero marginal products of outputs along the vertical and horizontal portions for respective inputs on the vertical and horizontal axes). A specified amount of output can only be produced by using one and only one combination of inputs. A change in the relative price of factor cannot lead to any substitution.

As result, MRTS and the elasticity of substitution both range between zero and infinity as the two extremes. As the degree of substitutability decreases, the degree of complementarity increases. For elasticities of substitution between zero and infinity, the isoquant is convex. Production occurs using both inputs which are substituted for each other based on their relative prices.

While MRTS measures the rate at which one factor can be substituted for another, in the production of output Y without changing the quantity of output or the rate at which the loss of certain units of one input will just be compensated for by additional units of another input at that point; the elasticity of substitution measures the degree of ease with which one factor is substituted for another or the technical similarity of factors of production.

With capital on the Y axis and labor on the X axis, the MRTS of labor for capital decreases along the isoquant, implying diminishing MRTS as continuous substitution of labor for capital occurs along a given isoquant. It is thus defined as the slope or gradient of the isoquant at a point which is equal to – ΔK/Δ L. Alternatively, it is given by the ratio of the marginal product of labor to that of capital. Mathematically, MRTS of labor (*i*) for capital (*j*) is given by,

$$\phi_{X_i Xj} = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}) \qquad (13)$$

In general terms, the marginal rate of substitution or elasticity of substitution of input X_i for input X_j is obtained by dividing the marginal product of input X_i by that of input X_j, thus,

$$\phi_{XiXj} = MRT_{X_i X_j} = \frac{\delta X_j}{\delta X_i} = \frac{MP_{YX_i}}{MP_{YX_j}} \qquad (14)$$

The decline in MRTS along a specific isoquant is referred to as the diminishing marginal rates of technical education (MRTE).

d) Elasticity of Substitution

The elasticity of substitution (ES) measures the strength of substitution effect and is defined as the proportionate change in the factor-proportions to the proportionate change in the marginal rate of technical substitution along a given isoquant. For two inputs labor (*L*) and capital (*K*), the elasticity of substitution of labor for capital (σ_{LK}) is given by

$$\sigma_{LK} = ES_{LK} = \frac{\%\Delta(L/K)}{\%\Delta(MP_K/MP_L)} = \frac{\%\Delta(L/K)}{\%\Delta MRTS_{K,L}} \quad (15)$$

Generally for inputs i and j, it is given by

$$\sigma_{ij} = ES_{ij} = \frac{\%\Delta(\frac{X_i}{X_j})}{\%\Delta(MP_j/MP_i)} = \frac{\%\Delta(\frac{X_i}{X_j})}{\%\Delta MRTS_{j,i}} = \frac{\Delta\ln(\frac{X_i}{X_j})}{\Delta\ln MRTS_{X_j,X_i}} \quad (16)$$

The elasticity of substitution defined above (σ_{ij}) shows the percentage change of the ratio the two inputs along an isoquant required to change the marginal rate of technical substitution by one percent. The two inputs are perfect substitutes if $\sigma_{ij} = \propto$ and not substitutable at all (perfect complements) if $\sigma_{ij} = 0$.

Using the above equation, the elasticity of substitution can be estimated using the equation below:

$$\ln(\frac{X_i}{X_j}) = \theta + \sigma_{ij}\ln MRTS_{X_j,X_i} \quad (17)$$

For the elasticity of labor (*i*) for capital (*j*), the equation can be rewritten as

$$\ln(\frac{L}{K}) = \theta + \sigma_{LK}\ln MRTS_{K,L} \quad (18)$$

Plotting the $\frac{X_i}{X_j}$ on the X-axis and the MRTS of X_J for X_i on the Y-axis, the elasticity of substitution increases from zero to infinity from the left of the substitution curve downwards towards the right.

For a production function, the elasticity of substitution measures the elasticity of the ratio of two inputs in terms of the ratio of their marginal products (or MRTS), thus it measures the curvature of an isoquant based on Hicks (1932) definition (Napasintuwong and Emerson, 2015), which indicates the substitutability between two inputs considered in the production process. Alternatively stated, it measures the effect of changes in the inputs, say capital and labor on the relative shares of the two inputs. The inverse of the elasticity of substitution measures the elasticity of complementarity (See Hicks, 1932, Mas-Colell, Whinston, Green, 2007, Stern, 2011 and Bergstrom, 2015).

Consequently, the elasticity of substitution can be used to classify three major categories of production functions - the linear, Constant elasticity of substitution (CES) and fixed proportion (piece-wise linear production function or Leontief) which are associated with $\sigma_{ij} = \propto$, $\sigma_{ij} = 0$, and $0 \prec \sigma_{ij} \prec \propto$, respectively. The Cobb-Douglas being a special case of the CES production function with $\sigma_{ij} = 1$.

3.2.4 Testing for the importance (significance) of the single input in the translog production function using the characteristic features of the translog function of a single factor of production

a) Single factor TL function and proper estimated parameters

The single factor translog production function in equation 19 is estimated using OLS for each of the factors, including capita and aggregate (and gender disaggregated labor) to test for their individual significance in equation 5 (equation 6).

$$\ln Y = \ln A_2 + \alpha_{2X_i} \ln X_i + (\frac{1}{2}).\beta_{2X_i}.\ln^2 X_i \quad (19)$$

Rewriting the estimated parameters in terms of the arithmetic means of the corresponding logarithms of the output and the input, equation 20 is obtained.

$$\ln A_2 = (\ln Y)_{med} - \alpha_k (\ln X)_{med} - (\frac{1}{2}).\beta_k.(\ln^2 X)_{med} \quad (20)$$

$$\alpha_{2X} = \alpha_{1X}.T_{\ln X} \quad (21)$$

$$\beta_{2X} = \beta_{1X}.T_{\ln^2 X} \quad (22)$$

Where

$(\ln Y)_{med}$ = arithmetic mean of the natural logarithms of the output indices (GDP)

$(\ln X)_{med}$ = arithmetic mean of the natural logarithms of the input (either total labor, capital, female labor or male labor depending on the single factor being investigated)

$(\ln^2 X)_{med}$ = arithmetic mean of the squares of the natural logarithms of the input

$T_{\ln X}$ = coefficient of alignment to collinearity hazard related to variable ln X

$T_{\ln^2 X}$ = coefficient of alignment to collinearity hazard related to variable $\ln^2 X$

$$T_{\ln X} \frac{1 - R(\ln X; \ln^2 X)r}{1 - R^2(\ln X; \ln^2 X)} \quad (23)$$

$$T_{\ln^2 X} = \frac{r - R(\ln X; \ln^2 X)}{r.(1 - R^2(\ln X; \ln^2 X)} \quad (24)$$

$$r = \frac{R(\ln Y; \ln^2 X)}{R(\ln Y; \ln X)} \quad (25)$$

r = the coefficient of correlation between the explanatory variables mediated by the resultative variable (Pavelescu [2010b]) and is only computed for the Pearson coefficient of correlation with the highest absolute value. For translog production function, r and $R(\ln X; \ln^2 X)$ usually have the same sign.

$R(\ln Y; \ln^2 X)$ = Pearson coefficient of correlation between the natural logarithm of output and the square of the natural logarithm of input X.

$R(\ln Y; \ln X)$ = Pearson coefficient of correlation between the natural logarithm of output and the natural logarithm of input X.

$$\alpha_{1X}^{C} = \frac{\operatorname{cov}(\ln Y; \ln X)}{D^2(\ln X)} \tag{26}$$

$$\beta_{1X^2}^{C} = \frac{2.\operatorname{cov}(\ln Y; \ln^2 X)}{\sigma^2(\ln^2 X)} \tag{27}$$

$Cov(\ln Y; \ln X)$ = covariance between natural logarithms of output and natural logarithms of input X.

$\sigma^2 \ln X$ = variance of natural logarithms of input X

$Cov(\ln Y; \ln^2 X)$ = covariance between natural logarithms of output and natural logarithms of input X.

$\sigma^2(\ln^2 X)$ = variance of the square of natural logarithms of input X

Following the definition of 'initial signal' and 'noise' used in Belsey (1991) and defining the parameters of the multiple regression as the 'noise' and those for the simple regression of the same single input (proper values) as the 'signal', equations 21 and 22 show that the 'noise' parameters are derived from the 'signal' parameters with the coefficient of collinearity hazard representing the ratio of the noise to signal (see Pavelescu, 2005, 2009, 2010a, 2011 for further discussion on the coefficient of collinearity)

The coefficients of alignment to collinearity hazard related to explanatory variables will be used to determine the main explanatory variable and secondary explanatory variables; and to identify and classify the collinearity that might occur in a multiple regression.

The parameters, α_{1X} and β_{1X^2} above represent the *proper estimated values of parameter* α *and* β *obtained* from equations 28 and 29, respectively; implying that the two equations can be used to estimate the proper estimated parameters.

$$\ln Y = \ln A_{1X} + \alpha_1 . \ln X \tag{28}$$

$$\ln Y = \ln A_{1X^2} + (\frac{1}{2})\beta_1 . \ln^2 X \tag{29}$$

b) Determination of Main and Secondary Explanatory Variables

If $|r| < 1$ and $T_{\ln X} > T_{\ln^2 X}$; $\ln X$ is the main explanatory variable and $\ln^2 X$ is the secondary explanatory variable; and if $|r| > 1$ and $T_{\ln X} < T_{\ln^2 X}$; $\ln X$ is the secondary explanatory variable and $\ln^2 X$ is the main explanatory variable.

c) Determination of the Type of Collinearity for each Explanatory Variable- Coefficient of Alignment to Collinearity Hazard

The extent of collinearity determines the feasibility of the TL function for multiple inputs. According to Pavelescu [2010b, 2011], the collinearity that can occur in multiple input analysis is categorized as either, *weak, degrading or harmful*; occurring: if all the coefficients of alignment to collinearity hazard are at least equal to 0.5; if all are positive and at least one of them is smaller than 0.5; and if at least one of them is negative, respectively. For two explanatory variables, collinearity can also be classified based on the relationship between the absolute values of r and $R(\ln X; \ln^2 X)$ (See Pavelescu, 2011 for further discussion of this approach). Existence of harmful collinearity makes the estimation completely unfeasible while degrading collinearity presents in forms of low and very low computed values of the standard student test.

d) Correlation between the Estimated Output Elasticity and Average Elasticity of Scale

As indicated by Pavelescu (2011), the estimated average elasticity of scale (E_{smed}) or equivalently the augmented output elasticity of scale for the single factor TL function for the entire period is an augmented elasticity of output related to the analyzed production factor. It given by

$$E_{smed} = \alpha_{1X} \cdot \left[\frac{1 - r.R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)} + 2\ln X_R \cdot \frac{\sigma(\ln X)}{\sigma(\ln^2 X)} \cdot \frac{r - R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)} \right] \quad (30)$$

$$E_{smed} = \alpha_{1X} \cdot \left[T_{\ln X} + 2\ln X_R \cdot \frac{\sigma(\ln X)}{\sigma(\ln^2 X)} . T_{\ln^2 X} \right] \quad (31)$$

Where

α_{1X} = the estimated proper elasticity of the output with respect to input X given by equation 28

$\ln X_R$ = the natural logarithm of the representative index of the input (K, L, L_F or L_m)

$$M_{TrX} = \left[\frac{1 - r.R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)} + 2\ln X_R \cdot \frac{\sigma(\ln X)}{\sigma(\ln^2 X)} \cdot \frac{r - R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)} \right] \quad (32)$$

$$= \left[T_{\ln X} + 2\ln X_R \cdot \frac{\sigma(\ln X)}{\sigma(\ln^2 X)} .rT_{\ln^2 X} \right]$$

= estimated translog multiplier

$\sigma(\ln X)$ = standard deviation of the logarithm of the employed input X

$\sigma(\ln^2 X)$ =standard deviation of the square of the logarithms of the employed input.

If the estimated average elasticity of scale is greater than (less than) than the output elasticity of a given factor, then the dynamic trajectory of that factor is conventionally under-exponential (over-exponential). The modeling factors influencing the average elasticity of scale are those which influence its components, that is the proper output elasticity and the translog multiplier.

e) Modeling factors for the estimated proper elasticity of the output for a factor input

The estimated proper elasticity is given by the expression in equation 33, implying three modeling factors: i) standard deviation the logarithm of the productivity of the input analyzed-the higher it is the larger the proper output elasticity; ii) the logarithm of the standard deviation of quantity employed of the employed quantity of input analyzed- the higher it is the smaller the proper output elasticity; and iii) Pearson correlation coefficient between the productivity and quantity of the analyzed employed input (the correlation between the input employed and the respective factor productivity- the higher it is the larger the output elasticity.

$$\alpha_{1X} = 1 + \frac{\sigma(\ln(Y/X)}{\sigma(\ln X)}.R(\ln(Y/X);\ln X) \quad (33)$$

Where

$R(\ln(Y/X);\ln X)$ = Pearson correlation coefficient between the productivity and quantity of the analyzed employed input.

$\sigma(\ln Y/X)$ = standard deviation of the logarithm of the productivity of the analyzed factor.

Thus to study, the average elasticity of scale for a given input in a single translog function, one should investigate modeling factors of the output elasticity and those of the estimated translog multiplier

Using equation 33 above, Pavelescu (2011), identified six possible correlations for the three modeling variables whereby output would increase for $\ln Y_R \succ 0, \alpha_{1X} \succ 1$ (decrease for $\ln Y_R \prec 0, \alpha_{1X} \succ 1$) following simultaneous increase (decrease) in quantity of input allocated and productivity of the input; increase for $\ln Y_R \succ 0, 0 \prec \alpha_{1X} \succ 1$(decrease for $\ln Y_R \succ 0, 0 \prec \alpha_{1X} \succ 1$) following an increase (decrease) in the quantity input allocated and decrease (increase) in the productivity of the input; and increase for $\ln Y_R \succ 0, \alpha_{1X} \prec 1$ (decrease for $\ln Y_R \prec 0, \alpha_{1X} \prec 1$) following a decrease (increase) in the quantity of input allocated and increase (decrease) in the productivity of the input, respectively.

Following Pavelescu (2011), equation 33 can be re-written in terms of three modeling factors, that is: i). ratio of the logarithm of the output and that of the production input; ratio of the coefficients of variation of the logarithm of the output and that of the input, which measures the characteristic features of the output relative to those of the input; and the Pearson coefficient of correlation between the logarithm of the output and that of the input which determines the degree of functionality between the two. The expression of α_{1X} in terms of these three modeling factors is indicated in equation 34.

$$\alpha_{1X} = \frac{\ln Y}{\ln X} . \frac{Cv(\ln Y)}{Cv(\ln X)} . R(\ln(Y; \ln X) \qquad (34)$$

Where

$Cv(\ln Y)$ = coefficient of variation of output (GDP)

$Cv(\ln X)$ = coefficient of variation of the logarithm of the analyzed factor input

$\ln Y$ = logarithm of output measured by GDP

$\ln X$ = logarithm of analyzed input employed

$R(\ln(Y; \ln X)$ = Pearson coefficient of correlation between the logarithm of the output and that of the input

If $R(\ln(Y; \ln X)$ =1, it implies $\frac{Cv(\ln Y)}{Cv(\ln X)} . R(\ln(Y; \ln X)$ =1, which in turn implies that $\alpha_{1X} = \frac{\ln Y}{\ln X}$ signaling a functional relationship between $\ln Y$ and $\ln X$ (the estimated output elasticity is a function of the ratio of the logarithms of the output and the respective input being analyzed).

The values of the modeling factors will be estimated and will be used to establish the link between each of the factors and the estimated proper elasticity of output for capital, aggregate and gender disaggregated (male/female) labor, as single inputs in the production process. The translog multiplier M_{TrX} is affected by several modeling factors (See Pavelescu [2011] for a discussion of these factors. The investigation of these factors is subject of another study.

e) Determination of Returns to Scale using the Single Input Translog Production Function

For single input function, the output elasticity is an indicator of the degree of returns to scale. Production experiences increasing, decreasing and constant returns to scale if the output elasticity is greater than 1, less than 1 and equal to 1, respectively. Returns to scale may change as the level of production changes.

3.3 Hypothesis tests

In order to choose between the TL function and the CD specification (or equivalently the constant returns to scale hypothesis), the null hypothesis (H_0) will be tested against the alternative (H_1), for all $i's$ and $j's$

$$H_0 : \beta_{ii} = \beta_{ij} = 0$$

$$H_1 : \beta_{ii} \neq \beta_{ij} \neq 0$$

The test can be performed using the F-test, with numerator degrees of freedom equal to the number of restriction for the Cobb-Douglas model and the denominator degrees of freedom equal to the sample size, n. Rejection of the null hypothesis signifies that the TL function is the appropriate model while failure to reject the null implies that the CD model is appropriate.

In case, the Cobb-Douglas function is rejected, additional tests for symmetry, constant returns to scale, weak separabilty and positivity will be performed on the TL function (See Khalil, 2005 for restrictions on parameters for these tests).

4. DATA

The analysis was done using the USA data, which was selected based on the availability of gender disaggregated data. It is ranked as having very high human development and high income per capita. The data used for analysis on GDP (entered in millions of 2010 U.S. dollars) was obtained from the World Development Indicators -WDI (2017); that for labor (both aggregate and gender disaggregated) was obtained from Organisation for Economic Co-operation and Development (OECD) Data- Labor force statistics (LSF); while that for capital stock was obtained from Feenstra, Inklaar and Timmer (2015) in millions of 2011 U.S. dollars. The base year for the capital stock data was adjusted from 2011 to 2010 (millions of 2010 U.S. dollars) to suit the base year for other data series obtained from WDI (2017). GDP is measured at market prices in millions (constant 2010 US$), labor is employment of either female or males of fifteen years and above (15+) while total employment (aggregate labor) is the number of those fifteen years above who are employed- includes all people ages 15+ who supply labor for the production of goods and services during a specified period. Female and male labor represent the number of people ages 15+ in those categories who supply labor for the production of goods and services during a specified period.

5. RESULTS AND DISCUSSION

5.1 Labor productivity results

The regression results in Table 1 show that an increase in the amount of capital in the economy per unit of aggregate labor does not significantly influence the productivity of aggregate labor (column 2), however, when the capital-labor ratio is disaggregated (Column 4), a 1% increase in the ratio of total capital to male labor decreases aggregate labor productivity by 1.98%; while a 1% increase in the ratio of total capital to female labor increases aggregate labor productivity by 25.45%. This implies that the more capital there is in the economy, the more productive the female labor force becomes, which in turn leads to a more productive aggregate labor force. Thus, in order to increase female labor productivity, more capital is required but this is not true for male labor in the USA.

Table 1: Aggregate\Gender Disaggregated Labor Productivity Functions

	Dependent variable: Aggregate Labor productivity $\ln \frac{GDPM}{L_T}$	
Constant	-1.9284 NS (0.4144)[1]	-67.9542***[2] (0.0000)
$\ln(K/l_M)$		-3.9985*** (0.0000)

$\ln^2(K/L_M)$		0.3987*** (0.0000)
$\ln(K/L_f)$		25.454*** (0.0000)
$\ln^2(K/L_f)$		-1.9777*** (0.000)
$\ln(K/l_T)$	1.1507 (0.1833)^{NS}	
$\ln^2(K/l_T)$	-0.0016 (0.9835)^{NS}	
R^{-2}	0.983	0.9907
Log-likelihood ratio	111.663	129.604
F-Stat	1560.959*** (0.0000).	1452.47*** (0.0000)

Notes to Table: *[1]Figures in parenthesis are probabilities. [2] The *, **, and *** imply significance at 10%, 5% and 1% level of significance. Source: Own estimation*

5.2 Cobb-Douglas, single and multiple input translog production function results

Table 2 presents the Cobb-Douglas and multiple input TL aggregate and gender disaggregated results. In order to determine the relevance of the multiple input TL parameters (determine the extent of multi-collinearity or significance of each of the inputs in the model), the single input TL were estimated and are presented in Table 3a and 3b. The results for the significance of the single input are presented in Table 4.

Table 2: Cobb-Douglas and Multiple Input Translog Aggregate and Gender disaggregated Results

	Dependent variable- $\ln GDP$				
	CD function			**TL- function**	
	Three input-Gender disaggregated model	**Two input -(Aggregate labor) model**		**Two-input-Aggregate labor model**	**Three input-gender disaggregated model**
Constant	-6.157*** (0.000)[1]	-3.123***[2] (0.0000)		102.3465 (0.0000)	-184.816*** (0.0000)
$\ln K$	0.871*** (0.0000)	0.8905*** (0.0000)		17.2174 (0.0013)***	35.378*** (0.0006)
$\ln l_f$	-0.1017** (0.0280)				-67.516*** (0.0000)
$\ln l_m$	0.755*** (0.0000)				45.166^{NS} (0.1782)
$\ln l_T$		0.3327*** (0.0005)		-42.1283*** (0.0001)	
$\ln^2 K$				1.0576** (0.0417)	1.1299** (0.0286)
$\ln^2 l_T$				5.2012*** (0.0062)	
$\ln^2 l_f$					-2.6299*** (0.0019)
$\ln^2 l_m$					-2.3504^{NS} (0.4655)
$\ln K \ln L_M$					-6.6296*** (0.0007)
$\ln K \ln L_f$					-0.0249^{NS}

					(0.9811)
$\ln K \ln l_T$				-4.543639** (0.0215)	
$\ln L_M \ln L_f$					11.2838*** (0.0000)
R^{-2}	0.9984	0.9972		0.9988	0.9995
LLF	138.649	123.6116		147.5959	172.411
F-Stat	10935*** (0.0000)	9669.25*** (0.0000)		8731.971*** (0.0000)	10990.66*** (0.0000)

Notes to Table**: [1]Figures in parenthesis are probabilities. [2] The *, **, and *** imply significance at 10%, 5% and 1% level of significance. General comment: The multi-input translog model cannot be interpreted (lacks relevance) due to high collinearity levels-harmful collinearity was identified for capital, aggregate labor and female labor.* ***Source: *Own estimation*

Table 3A: Single Input Translog Production Function Results for Capital and Aggregate Labor

Dependent variable- $\ln GDP$							
	Capital				**Aggregate labor**		
Constant	6.8656^{NS} (0.2193)	-2.3258*** (0.0000)	6.7424*** (0.0000)		85.4416*** (0.0007)	-6.6147*** (0.0000)	4.5769 (0.0000)
$\ln K$	-0.0145^{NS} (0.9823)	1.0691*** (0.0000)					
$\ln l_T$					-14.0832*** (0.0012)	1.9502*** (0.0000)	
$\ln^2 K$	0.0319^{NS} (0.1019)		0.0315*** (0.0000)				
$\ln^2 l_T$					0.6978***		0.0849***

					(0.0003)		(0.0000)
R^{-2}	0.9966	0.9965	0.996709		0.9838	0.9795	0.9806
LLF	118.4994	117.071	118.4991		75.256	68.1725	69.6379
F-Stat	8024*** (0.0000)	15526.8*** (0.0000)	16357.5*** (0.0000)		1644.71*** (0.0000)	2579.31*** (0.0000)	2723.39*** (0.0000)

Notes to Table: *[1]Figures in parenthesis are probabilities. [2] The *, **, and *** imply significance at 10%, 5% and 1% level of significance.* ***Source:*** *Own estimation*

Table 3B: Single Input Translog Production Function Results for Male Labor and Female Labor

Dependent variable- $\ln GDP$							
	Male labor				**Female labor**		
Constant	-13.4574^NS (0.8008)	-15.23*** (0.0000)	0.3009*** (0.0000)		70.0828*** (0.0000)	1.7924*** (0.0000)	8.7477*** (0.0000)
$\ln l_f$					-11.6197*** (0.0000)	1.3201*** (0.0000)	
$\ln l_m$	2.5100^NS (0.7964)	2.8338*** (0.0000)					
$\ln^2 l_f$					0.6122*** (0.0000)		0.0626*** (0.0000)
$\ln^2 l_m$	0.0148 (0.9735		0.12924*** (0.0000)				
R^{-2}	0.9779	0.9783	0.9787		0.9817	53.0225	0.967828
LLF	66.6389	66.6383	66.6034		71.8321	0.9644	55.79794
F-Stat	1195.283*** (0.0000)	2436.5*** (0.0000)	2433.324*** (0.0000)		1449.126*** (0.0000)	1464.33*** (0.0000)	1625.466*** (0.0000)

Source: *Own estimation*

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The results obtained for equation 19 (Table 3a and 3b summarized in Table 4, rows 1), reveal that for all the variables (capital, total labor and female labor) where a negative coefficient (negative output elasticities) for $\ln X$ was obtained (violation of expected sign), it was discovered that $\ln X$ was the secondary explanatory variable, with $\ln^2 X$ being the primary explanatory variable (Table 5) implying negative corresponding marginal products, thus lack of positivity of the functions. This implies that the corresponding elasticity of scale α_{mi} for capital, aggregate labor and female labor (Table 4, row5) are bound to be inaccurate. Positivity was satisfied for the male labor Single factor TL two variables function, where the expected sign for $\ln X$ was obtained and $\ln X$ rather than $\ln^2 X$ was the main explanatory variable, however, the two coefficients were non-significant while those for the corresponding proper estimated output elasticities obtained using equations 20 and 21 of α_{1k}^{e} =2.8338 and $\frac{1}{2}\beta_{2x} = \varphi_{ij}$ = 0.1292, (Table 4 rows 9 and 10, column 4) respectively, were positive and and significant. Thus, the model fails to estimate the proper parameters.

Table 4: Testing for the Importance (Significance) of the Single Input in the Translog Function

Single factor translog estimated parameters based on equation 19				
	$X_R = K$	$X_R = L_T$	$X_R = L_M$	$X_R = L_f$
Estimated α_{2x}	-0.0145^{NS}	-14.0832***	2.510^{NS}	-12.05***
Estimated $\frac{1}{2}\beta_{2x} = \varphi_{ij}$	0.0319*	0.6978***	0.0148^{NS}	0.6328***
Estimated $\beta_{2x} = 2\varphi_{ij}$	0.0638	1.3956	0.0296	1.2656
$\ln X_R$	17.3162	11.699	11.0881	10.9152
$\alpha_{mi} = \frac{\delta \ln Y}{\delta \ln X_i} = \alpha_i + \sum_{j=1}^{n} \varphi_{ij} \ln X_j$	0.537887	-5.9196	2.6741	-5.1428
Estimated proper output elasticities (α and β) obtained using equation 20 and 21				
	$X_R = K$	$X_R = L_T$	$X_R = L_M$	$X_R = L_f$

α_{1X}	1.0691***	1.9502***	2.8338***	1.3201***
$\frac{1}{2}\beta_{1X}$	0.0315***	0.0849***	0.1292***	0.0626***
Computed values of $T_{\ln X}$ and $T_{\ln^2 X}$ based $\alpha_{2X} = \alpha_{1X}.T_{\ln X}$ and $\beta_{2X} = \beta_{1X}.T_{\ln^2 X}$ and equation 9 and 10				
	$X_R = K$	$X_R = L_T$	$X_R = L_M$	$X_R = L_f$
$T_{\ln X}$	-0.01356	-7.22141	0.885736	-9.12802
$T_{\ln^2 X}$	0.030926	8.219081	0.114551	10.1248
$T_{\ln X} = \frac{1-R(\ln X;\ln^2 X)r}{1-R^2(\ln X;\ln^2 X)}$	-0.01273	-7.10538	0.893097	-8.83062
$T_{\ln^2 X} = \frac{r-R(\ln X;\ln^2 X)}{r.(1-R^2(\ln X;\ln^2 X))}$	1.512669	8.601078	0.606909	10.31361
Computed proper output elasticities based on equation 12 and 13				
	$X_R = K$	$X_R = L_T$	$X_R = L_M$	$X_R = L_f$
$\alpha_{1X}^C = \frac{\text{cov}(\ln Y;\ln X)}{D^2(\ln X)}$	1.049493	1.914796	2.782349	1.296104
$\beta_{1X^2}^C = \frac{2.\text{cov}(\ln Y;\ln^2 X)}{\sigma^2(\ln^2 X)}$	0.061827	0.166746	0.253782	0.122835
$\frac{1}{2}\beta_{1X^2}^C = \frac{\text{cov}(\ln Y;\ln^2 X)}{\sigma^2(\ln^2 X)}$	0.030913	0.083373	0.126891	0.061418

Source: *Own estimation and computation*

The results in Table 5 revealed the existence of harmful collinearity for capital, female labor and total labor functions while that for male labor had either weak collinearity or degrading collinearity based on computed or estimated values of $T_{\ln X}$ and $T_{\ln^2 X}$. *The existence of harmful collinearity for aggregate labor and/or female labor and capita implies that* the three input (male labor, female labor and capital) and two input (capital and total labor) TL functions in Table 2 are inaccurate and should not be used for further interpretation.

With, this in mind, further discussions on the TL functions are (proper parameters) based on the single variable single input TL estimates (proper parameters) as well as the corresponding computed values based on the modeling factors (Sub-section 5.3).

Table 5: Determination of Main/Secondary Explanatory Variable and Type of Collinearity

	Capital	**Aggregate Labor**	**Male Labor**	**Female Labor**
$R(\ln X;\ln^2 X)$[1]	0.9999	1.0000	1.0000	0.9999
$R^2(\ln X;\ln^2 X)$	0.9998	0.9999	1.0000	0.9998
$R(\ln Y;\ln^2 X)$	0.9984	0.9904	0.9893	0.9841
$R(\ln Y;\ln X)$	0.9983	0.9899	0.9893	0.9824
$r = \frac{R(\ln Y;\ln^2 X)}{R(\ln Y;\ln X)}$	1.0001	1.0005	1.0000	1.0017
$T_{\ln X} = \frac{1-R(\ln X;\ln^2 X)r}{1-R^2(\ln X;\ln^2 X)}$	-0.0127	-7.10538	0.8931	-8.8306
$T_{\ln^2 X} = \frac{r-R(\ln X;\ln^2 X)}{r.(1-R^2(\ln X;\ln^2 X))}$	1.5127	8.6011	0.6069	10.3136
Main explanatory variable[2]	$\ln^2 K$	$\ln^2 L_t$	$\ln L_m$	$\ln^2 L_f$
Secondary explanatory variable	$\ln K$	$\ln L_t$	$\ln^2 L_m$	$\ln L_f$
Type of collinearity detected	Harmful	Harmful	Weak or degrading [3]	Harmful

NOTES TO TABLE: *[1] As expected* $R(\ln X;\ln^2 X)$ *and* r *have the same sign for all the inputs considered. [2]If* $|r|<1$ *and* $T_{\ln X} > T_{\ln^2 X}$*;* $\ln X$ *is the main explanatory variable and* $\ln^2 X$ *is the secondary explanatory variable; and if* $|r|>1$ *and* $T_{\ln X} < T_{\ln^2 X}$*;* $\ln X$ *is the secondary explanatory variable and* $\ln^2 X$ *is the main explanatory variable. Weak collinearity occurs if* $T_{\ln X} \geq 0.5$ *and* $T_{\ln^2 X} \geq 0.5$*; degrading collinearity occurs if* $T_{\ln X}$ *and* $T_{\ln^2 X}$ *are both positive and at least one of them is smaller than 0.5; while harmful collinearity* *occurs if at least $T_{\ln X}$ or $T_{\ln^2 X}$ is negative.* [3] *The single factor equation for male labor was associated with weak collinearity based on computed values but with degrading collinearity based on estimated values.* ***Source:*** *Own computation*

5.3 Indicators based on single variable single input translog model

5.3.1 Proper output elasticities

Table 6 shows that the average proper output elasticity ((elasticity of scale) (see Table 6, row 4- average for computed and estimated values), for capital, total labor, male labor and female labor of 1.065, 1.944, 2.827 and 1.313, respectively. They are all greater than 1, implying increasing returns to scale for all the single inputs.

Table 6: Summary Elasticities Based on Estimated and Computed Values

Computed/estimated output elasticities	**Input**			
	$X_R = K$	$X_R = L$	$X_R = L_M$	$X_R = L_f$
estimated α_{1X}	1.069*** (0.0086)[1]	1.950*** (0.0384)	2.8338*** (0.0574)	1.3201*** (0.0345)
$\alpha_{1X}^C = \frac{\text{cov}(\ln Y; \ln X)}{D^2(\ln X)}$	1.0495	1.9148	2.7823	1.2961
$\alpha_{1X}^{cv} = \frac{\ln Y}{\ln X} \cdot \frac{Cv(\ln Y)}{Cv(\ln X)} . R(\ln(Y; \ln X)$	1.0715	1.9594	2.8588	1.3155
$\alpha_{\alpha_{1X}}^P = 1 + \frac{\sigma(\ln(Y/X)}{\sigma(\ln X)} . R(\ln(Y/X); \ln X)$	1.0691	1.9505	2.8338	1.3201
$\alpha_{1X} ave^2$	**1.0648**	**1.9437**	**2.8272**	**1.3129**
E_{smed}	**120.3649**	**1538.802**	**221.7859**	**791.3006**

***Notes to Table**: [1]Figures in parenthesis are standard errors. [2]$\alpha_{1X}ave$ implies average proper output elasticity (elasticity of scale for single input. **Source:** Own estimation and computation*

The average output elasticity for capital of 1.0648 though greater than 1, is only 54.78%, 37.66% and 81.10% that for total labor, male labor and female labor, respectively. This implies that total labor is about twice (1.825 times) as productive as capital in the USA, with male labor being 2.15 times as productive as female labor (female labor is less productive than male labor), implying that gross domestic product in the U.S. is more dependent on labor employment compared to capital; and male labor to be specific, followed by female labor and capital in that order.

Given the current capital outlays, as indicated earlier, aggregate productivity, thus aggregate output, would increase much more if the productivity of female labor force is boosted to the level of the male labor force.

The lower productivity of women in the economy may be explained first by the several factors including but not limited to the fact that women carry the triple burden of production, reproduction and community management which may prevent them from offering as much labor time, investing in human capital, health and education, accessing finance for capital formation, among other challenges; and second by the fact that most of the activities undertaken by women are not recognized in the national income accounts, implying undercounting of economic activity and the contribution of women to national output. This calls for measures that can be used to address the gender challenges.

5.3.2 Correlation between logarithm of output representative index, output estimated proper elasticity and production factor allocated quantity and productivity

The representative index of logarithm of GDP (16.185) is greater than zero, and the proper output elasticities of capita, aggregate/male/female labor are all implying that output would increase following a simultaneous increase in quantity allocated and productivity of each of the factors investigated. This justifies the need to increase productivity and labor participation rates for both male and female labor as well as that for capital, with the ultimate aim of increasing overall output.

As illustrated in Figure 1, the labor force participation rates for men have declined from 89.8% in 1960 to 78.5% in 2015 while that for women increased from 42% in 1960 to 66.9%; but the gap between the two (47.8% in 1960 and 11.6%in 2015) has not yet been closed. Since the labor force participation rates have persistently lower for women compared to those for men throughout the period of analysis, significant results would be achieved if the labor force participation rates of women are increased by addressing the obstacles that hinder women from engaging in or offering sufficient labor time in the labor market. The employment/population ratios of 39.5% and 85.2% in 1960 and 58.7% and 74% for women and men, respectively, further underscore the need to increase the number of women employed.

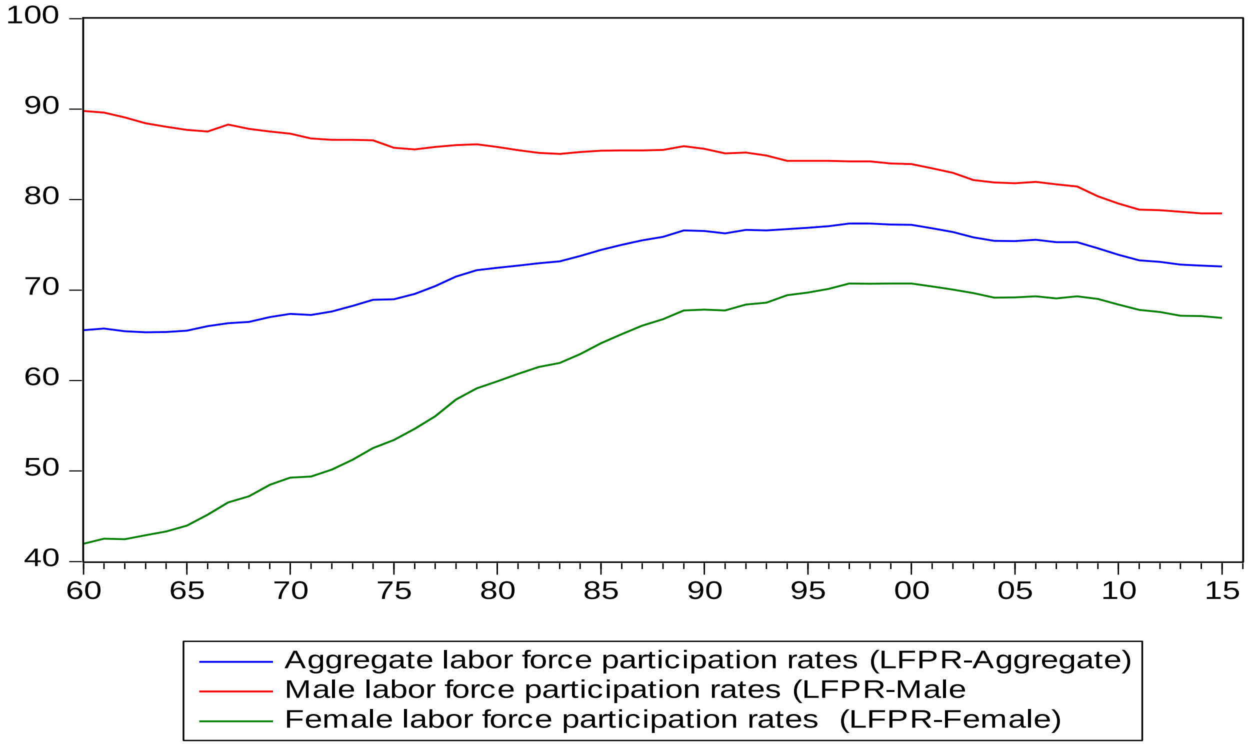

Figure 1 Aggregate/Gender Disaggregated Labor Force Participation Rates in the USA

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5.3.4 Correlation between the estimated values of output elasticity and average elasticity of scale for a single factor translog production function

The estimated augmented output elasticity (average elasticity of scale) obtained by the product of the estimated proper output elasticity and the translog multiplier (equation 30, see Pavelescu 2011 for detailed discussion) presented in Table 7, show that in all cases, the augmented single input augmented output elasticity (120.36, 1538.80, 221.79 and 791.3 for capital, aggregate labor, male labor and female labor, respectively) is greater than the corresponding single input average proper output elasticity (1.065, 1.944, 2.827, and 1.313, respectively), implying that the dynamic trajectory of each of the production factors is conventionally under-exponential.

Table 7: Computation of Average Elasticity of Scale

Formular	$X_R = K$	$X_R = L_T$	$X_R = L_M$	$X_R = L_f$
$\ln X_R$	17.3162	11.699	11.0881	10.9152
$R(\ln X; \ln^2 X)$	0.9999	1.0000	1.0000	0.9999
R	1.0001	1.0005	1.0000	1.0017
$R^2(\ln X; \ln^2 X)$	0.9998	0.9999	1.0000	0.9998
$\sigma(\ln X)$	0.4653	0.2529	0.1740	0.3708
$\sigma(\ln^2 X)$	0.2165	0.0640	0.0303	0.1375
$\dfrac{\sigma(\ln X)}{\sigma(\ln^2 X)}$	2.1492	3.9540	5.7488	2.6969
$\dfrac{1 - r.R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)} = T_{\ln X}$	-0.0127	-7.1054	0.8931	-8.8306
$\left[\dfrac{r - R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)}\right] = r.T_{\ln^2 X}$	1.5128	8.6057	0.6069	10.3315
$2\ln X_R . \dfrac{\sigma(\ln X)}{\sigma(\ln^2 X)} . \dfrac{r - R(\ln X; \ln^2 X)}{1 - R^2(\ln X; \ln^2 X)}$	112.598	796.1534	77.3714	608.2554
M_{TrX}	112.5853	789.0481	78.26447	599.4247

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estimated α_{1X}	1.0691***		1.9502***		2.8338***		1.3201***
Average elasticity of scale (E_{smed}) or Augmented output elasticity	120.3649		1538.802		221.7859		791.3006

The augmented output elasticity of scale of 791.3 for female labor, is about 3.568 times that of the male labor force of 221.7859 (see Table 7), implying that employing more women compared to men would result in more output produced. Also, the augmented output elasticity for capital of 120.365 is only 7.82%, 54.27% and 15.21% of the elasticity of scale for total labor, male labor and female labor, respectively; implying that increasing the average scale of employment of labor is, in general, more productive than increasing the average scale of employment of capital. To gain an understand underlying factors for the average elasticity of scale each of inputs in a single translog function, one should investigate the modeling factors of the output elasticity and those of the estimated translog multiplier, however, this paper focused on only the factors of output elasticity, presented in the next sub-section.

5.3.5 Modeling factors for output elasticity for capital, aggregated labor and gender disaggregated labor inputs based on productivity of the input (equation 33)

The results based on equation 33 in Table 8, show that the lower proper output elasticity of capital compared to labor is due to the lower standard deviation of the logarithm of the productivity of the capital relative to the logarithm of the standard deviation of the quantity of capital employed and the lower Pearson correlation coefficient between its productivity and quantity compared to those of the labor categories investigated. Male labor has a higher proper output elasticity compared to female labor due to relatively higher standard deviation of the logarithm of its productivity-higher variability in productivity of the different levels of inputs utilized, a lower standard deviation in the logarithm of the number of male labor employed-lower variations in the number male labor employed, and a higher correlation between number of male labor employed and the productivity of male labor -higher Pearson correlation coefficient between the productivity and quantity of the analyzed employed input compared to their female counterparts: 0.975 and 0.787 for male labor and female labor, respectively.

Table 8: The Modeling Factors for Output Elasticity for Capital, Aggregate Labor, and Gender Disaggregated Labor Inputs Based on Productivity of the Input (equation 33)

	$X_R = K$	$X_R = L$	$X_R = L_M$	$X_R = L_f$
$\sigma(\ln Y/X)$	0.0433	0.2505	0.3272	0.1509
$\sigma(\ln X)$	0.4653	0.2529	0.1740	0.3708
$R(\ln(Y/X);\ln X)$	0.7418	0.9593	0.9750	0.7868
$\frac{\sigma(\ln(Y/X))}{\sigma(\ln X)}$	0.0931	0.9907	1.8809	0.4069
$\alpha^{P}_{\alpha_{1X}} = 1 + \frac{\sigma(\ln(Y/X))}{\sigma(\ln X)}.R(\ln(Y/X);\ln X)$	1.0691	1.9505	2.8338	1.3201

Partial Proper Output elasticity for input X (average based on different formulars)[1]

Input	$\alpha_{1X}\, ave$
Capital	1.0648
Aggregate labor	1.9437
Male labor	2.8272
Female labor	1.3129
$\ln Y_R$	16.1847

Notes to Table: *[1]All the individual input have partial output elasticities that are greater than one implying increasing returns to scale.* ***Source:*** *Own estimation*

Alternatively, based on equation 34, (Table 9), the lower output elasticity of capital compared to that for other inputs is the consequence of the lower ratio of the logarithm of output and the capital utilized of 0.9347 compared to the corresponding ones of 1.383, 1.46, and 1.483 for the labor categories. This implies more capital intensive technologies are used in the USA, thus lower productivity of the capital compared to labor. This signals the fact that capital formation should be emphasized in those areas where production is currently less capital intensive. Since women world over, have less access to capital compared to men, this would imply that capital formation would be more productive if injected in the production activities majorly undertaken by women. This would not only increase the productivity of capital but will also increase the productivity of women and the aggregate labor.

On the other hand, the output elasticity of female labor is much lower than that for male labor and aggregate labor, due to the lower ratio of coefficient of variation of logarithm of output to that of female labor employed of 0.9031 compared to 1.4308 and 1.9797 for aggregate labor and male labor respectively. These results imply that the spread (amount of variability relative to the mean) for output is smaller than that for female labor, but is bigger (ratios greater than one) than that for male labor and aggregate labor. This implies that lower productivity of female labor compared to the male labor is due to the higher variability of the female labor force relative to output compared to the corresponding one for their male counterpart. Thus, to increase the productivity of the female labor, it is necessary to address the factors that cause high variability in the female labor force. This could be the consequence of having more women undertaking part-time jobs, being laid off more frequently compared to their male counterparts, resigning jobs to accompany their families when their husbands are relocated, among other reasons.

Table 9: The Modeling Factors for Average Output Elasticity Scale for Capital, Aggregated Labor, and Gender Disaggregated Labor Inputs Based on Estimated Output Elasticity (equation 34)

	$X=K$	$X=L_T$	$X=L_M$	$X=L_f$
$\ln X_R$	17.3162	11.699	11.0881	10.9152
$Cv(\ln X)=$	2.7310	2.1919	1.5841	3.4726
$R(\ln(Y;\ln X)$	0.9983	0.9898	0.9893	0.9823
$\frac{\ln Y}{\ln X}$	0.9347	1.3834	1.4596	1.4828
$\frac{Cv(\ln Y)}{Cv(\ln X)}$	1.1483	1.4308	1.9797	0.9031
$\alpha_{1X}^{cv}=\frac{\ln Y}{\ln X}.\frac{Cv(\ln Y)}{Cv(\ln X)}.R(\ln(Y;\ln X)$	1.0715	1.9594	2.8588	1.3155
$\ln Y_R$ =16.1847 and $Cv(\ln Y)$ =3.1361				

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5.3.6. *Marginal products of aggregate\gender disaggregated labor and capital based on partial output elasticities*

The marginal product for each of the inputs was computed using the proper partial output elasticity estimated using equation 28, by multiplying the proper partial output elasticity by the output-input ratio. All the marginal products for all the four inputs are positive implying that the function is well behaved. The marginal product of capital on average of 0.339 million dollars worth of output has been relatively constant (almost zero trend coefficient of 0.000694, mean of 0.339 ± 0.014 and a coefficient of variation of 4.3%) implying that the productivity of capital has remained fairly constant over the period as illustrated in Figure 2.

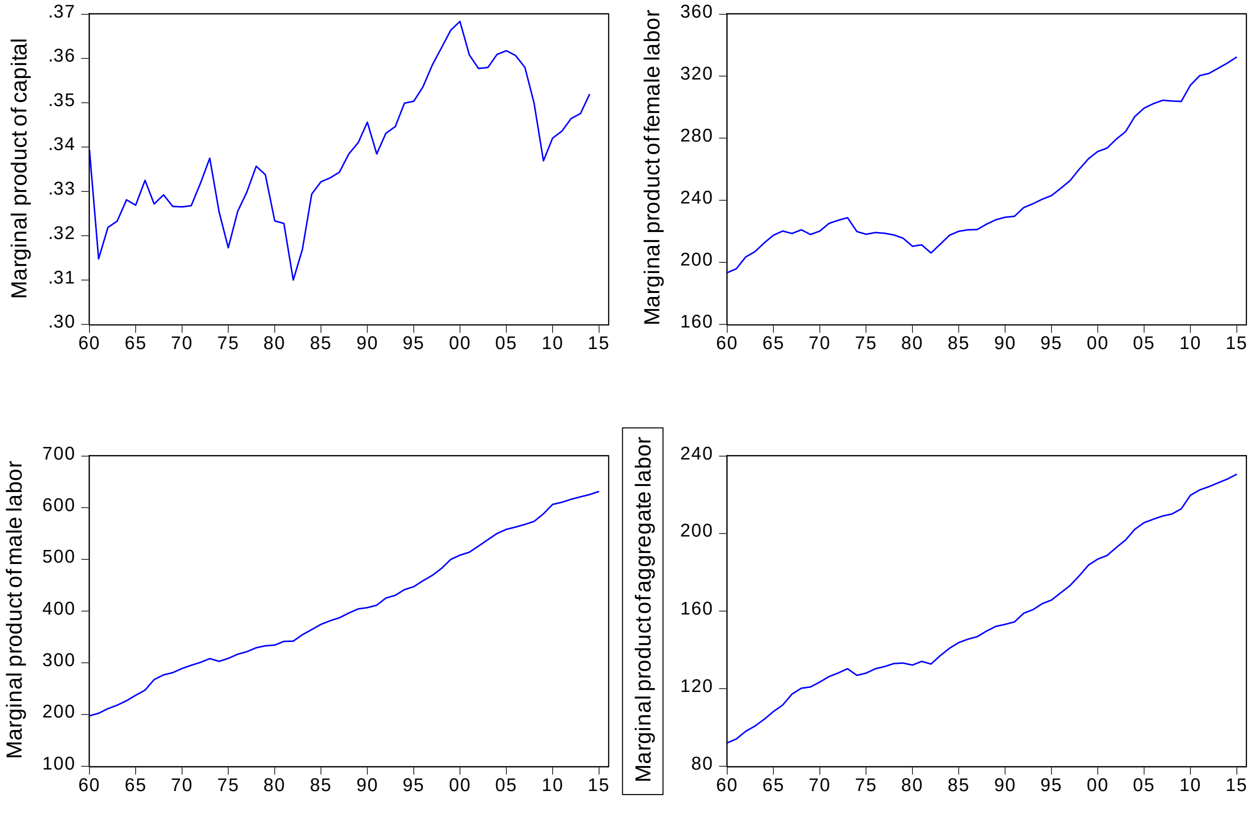

Figure 2 Trends of Marginal Product of Aggregate/Gender Disaggregated Labor and Capital

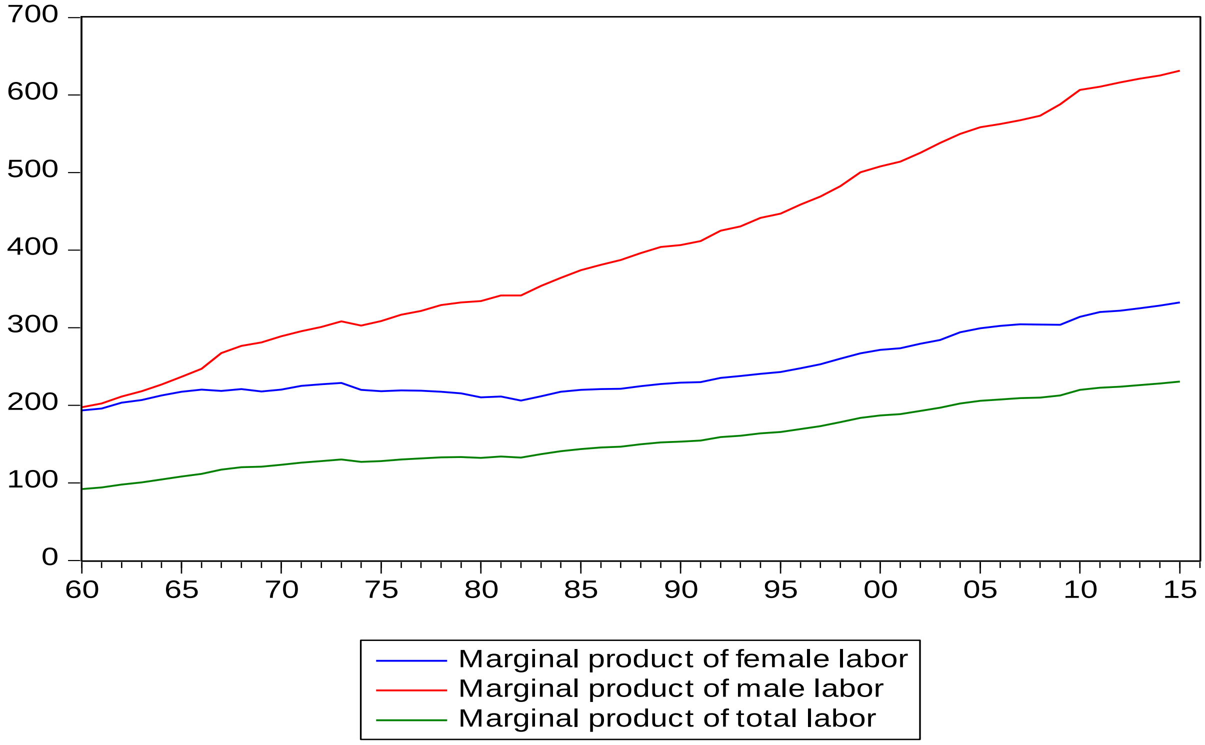

Figure 3. Comparison of Marginal Products of Labor

Although the marginal product (on average $403.39 worth of output) for male labor has over the years been higher than that for female labor (on average $244.67 worth of output), which in turn has been higher than the marginal product of the aggregate labor (on average $155.74.39 worth of output), all the three have had an upward trend, with significant (probabilities of zero) trend coefficients of 7.8503, 2.1807, and 2.3896, respectively. This implies that the growth in the productivity of female labor has been on average less than that for male labor (yet they were nearly equally productive in 1960 with marginal product of 193.25 and 197.37 million dollars worth of output) and that for aggregate labor. This lower productivity of female labor may be explained by the fact that women have the heavy burden of performing the triple gender roles of production, reproduction and community management roles simultaneously while men primarily undertake productive and community politics activities. Further, given the coefficients of variation of 24.98%, 31.36% and 15.88% for the marginal product of total, male and female labor, respectively, it can be concluded that variations in labor productivity was twice as much that for male labor compared to female labor.

Low growth in the female labor productivity has caused that for aggregate labor to be lower than that for male labor. Also, if labor is paid the value of its marginal product, these results would signal the fact that female labor is paid a lower wage compared to male labor (presence of a gender wage gap), which has a relatively higher marginal product. Attaining gender equality in terms of eliminating the gender wage gap-attaining wage parity for male and female labor- would thus require interventions that can increase the productivity of female labor relative to male labor.

Compared to capita, labor of all categories has a much higher marginal product, implying that capita is in much abundant supply compared to labor while labor is scarce.

5.3.7 Marginal rates of technical substitution between inputs-aggregate/gender disaggregated labor and capital

The marginal products obtained above, were used to compute the marginal rates of technical substitution (MRTS) for different inputs. The results show that MRTS of aggregate labor, male labor and female for capital are on average 456.24, 1179.24 and 719.04, respectively, with corresponding variances (and coefficient of variation) of 10053.65, (21.98%%), 112510.1 (28.44%), and 8758.33(13.015%). This implies that one unit of aggregate labor, male labor and female labor can be substituted for 456.24, 1179.24 and 719.04 units of capital. Figure 4a and 4b show that the marginal rate of technical substitution of labor for capital has increased over the years for all categories considered from 271.25 to 647.99, 581.66 to 1776.60 and 569.54 to 933.22 with significant trend coefficients of 6.103623, 20.72292 and 4.923703 (probabilities of zero) for aggregate labor, male labor and female, respectively. This implies that labor, regardless of category considered has been relatively more productive that capital, with greater increase in the relative productivity of male labor to capital compared to that for female labor (and total labor) to capital.

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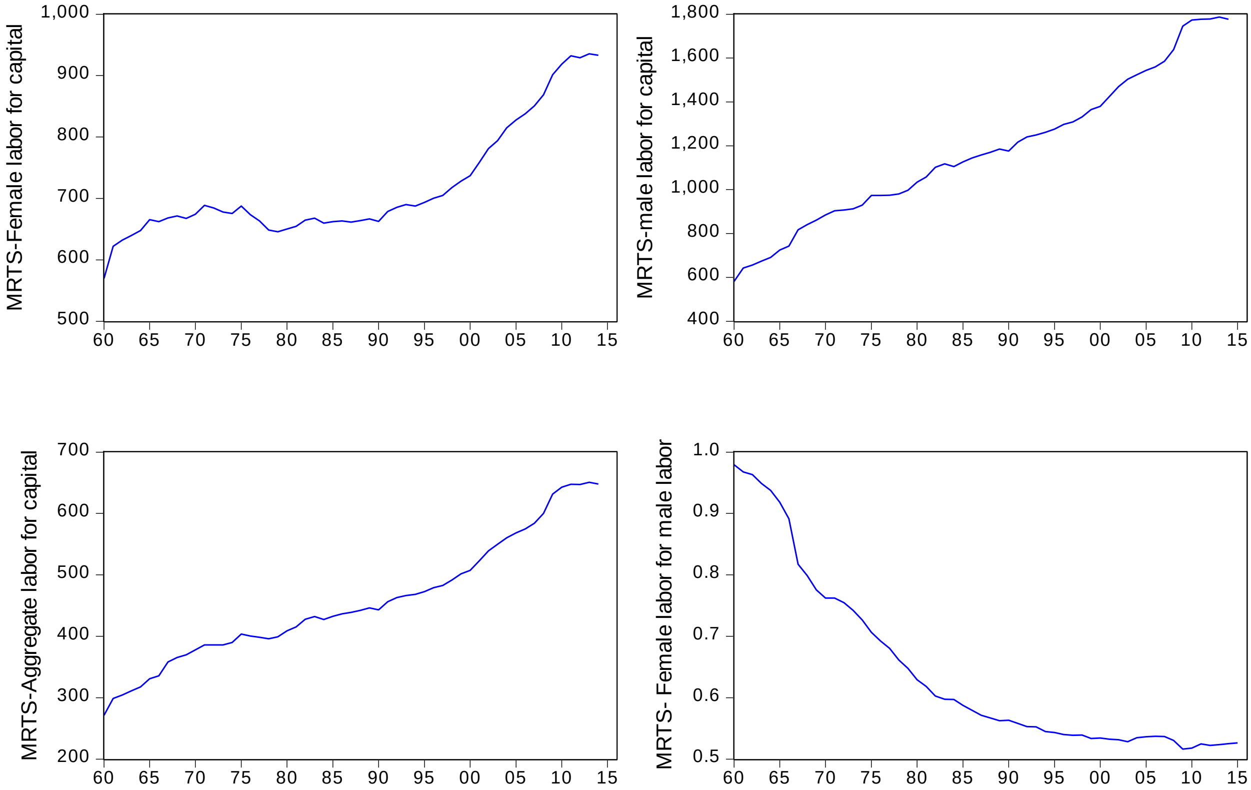

Figure 4a. Marginal rate of technical substitution Labor Categories for Capital and Female Labor for Male

Also, based on the coefficient of variation, the variations in the MRTS of male labor for capital were more than twice those for the MRTS of female labor for capita. Although the MRTS for female and male labor for capital were nearly the same in 1960 at 569.476 and 581.734 that for men increased to 1776.602 while that for female only increased to 933.225. The increase in productivity of labor in general could be due to human capital development and/or technology advancement, among other factors; this has not yielded gender parity in terms of relative productivity the gender disaggregated labor to capital. This calls for gender sensitive measures aimed at increasing the relative productivity of the female labor forces with the aim of attaining parity with the male counterparts. This calls for investigation of the factors influencing the productivity of gender disaggregated labor.

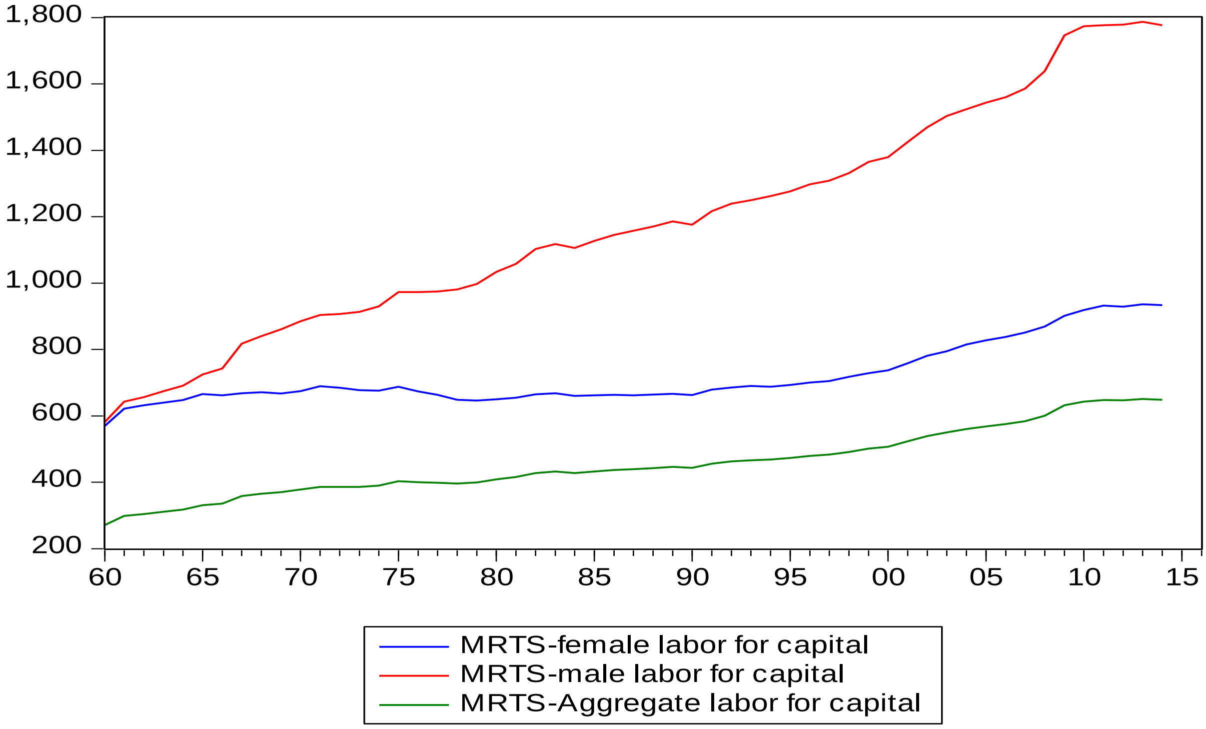

Figure 4b. Comparison of Marginal Rate of Technical Substitution of Labor (Aggregate, Female and Male) for Capital

The marginal rate of technical substitution of female labor for male labor is on average 0.6444, with variances (and coefficient of variation) of 0.02033 (22.126%). This implies that one unit of female labor can on average be substituted for 0.644 ($\pm$0.143) units of male labor. The Figure –shows that the marginal rate for technical substitution of female labor for male labor has declined over the years from 0.98 to 0.53 with significant negative trend coefficients of -0.008(Probability 0.0000). This implies that female labor has become relatively less productive compared to male labor. In the earlier period, 1960-1962, female labor was almost as productive as male labor (could be substituted one for one-MRTS of 0.98 to 0.96) but from 1995 to 2014, female labor is only about as half as productive as male labor (can be substituted two female labor units for one male labor unit-MRTS of 0.543 to 0.526) in the production process.

It is however, important to note that although the range of the ratio of female to male labor was 0.4757 to 0.9024, with an average of 0.75, implying that the number of women employed relative to the number of men employed has increased over the period from less than half to up to 90.24%, (parity not attained at all over the period). The increase in the ratio could be ascribed to the high development index and the affirmative action efforts over the years. This increase in the female labor relative to male labor should be coupled with increase in the female labor productivity, thus efforts towards both gender parity in numbers as well as productivity are required.

5.3.8 Elasticity of substitution

The elasticity of substitution for the different input pairs were estimated using equation 17. As shown in Table 10, the elasticity of substitution of aggregate labor, male labor and female labor for capital were 1.82, 2.651, and 1.235, respectively. These results indicate that capital and labor regardless of the category considered are neither perfect substitutes nor perfect complements. The substitutability between male labor and capital is greater than that between female labor and capital, thus the degree of complementarity of 0.8097 between female labor and capital (inverse of the elasticity of substitution) is greater than that between male labor and capital of 0.3772. The elasticity of substitution of aggregate labor for capital (thus the degree of complementarity) ranges between that for male labor and female labor.

Table 10**:** Elasticity of Substitution for Input Pairs

	Elasticity of substitution			
	i=female labor j = capital	i = male labor j = capital	i= total labor j = capital	i = female labor j = male labor
Cons.	-1.52E-17 (0.0000)	-1.22E-17 (0.0000)	2.99E-17 (0.0000)	-5.10E-15 (0.0000)
σ_{ij}	1.235 (0.0000)	2.651 (0.0000)	1.824151 (0.0000)	0.4658 (0.0000)

Source: *Own estimation*

Graphical analysis reveals that although the ratio of aggregate labor to capital (Figure 5) and the MRTS of capital for labor, both aggregate and gender disaggregated (Figure 6), declined over the 1960 to 2014 period, implying that more capital intensive technologies have been adopted, the ease of substitution (elasticity of substitution given by the slope of the graph of the logarithm of the labor-capital ratio versus the logarithm of the MRTS of capital for labor (see Figure 7a to 7c, based on equation 18) between labor and capital has remained constant for each labor category.

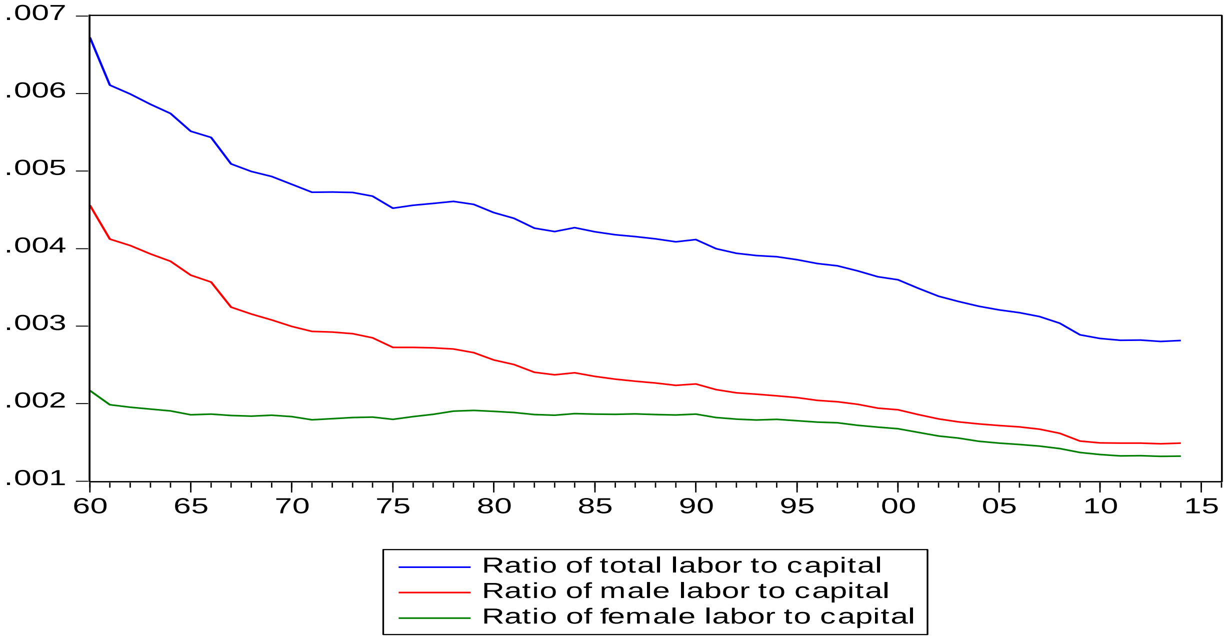

Figure 5. Ratio of Aggregate/Gender Disaggregated Labor to Capital

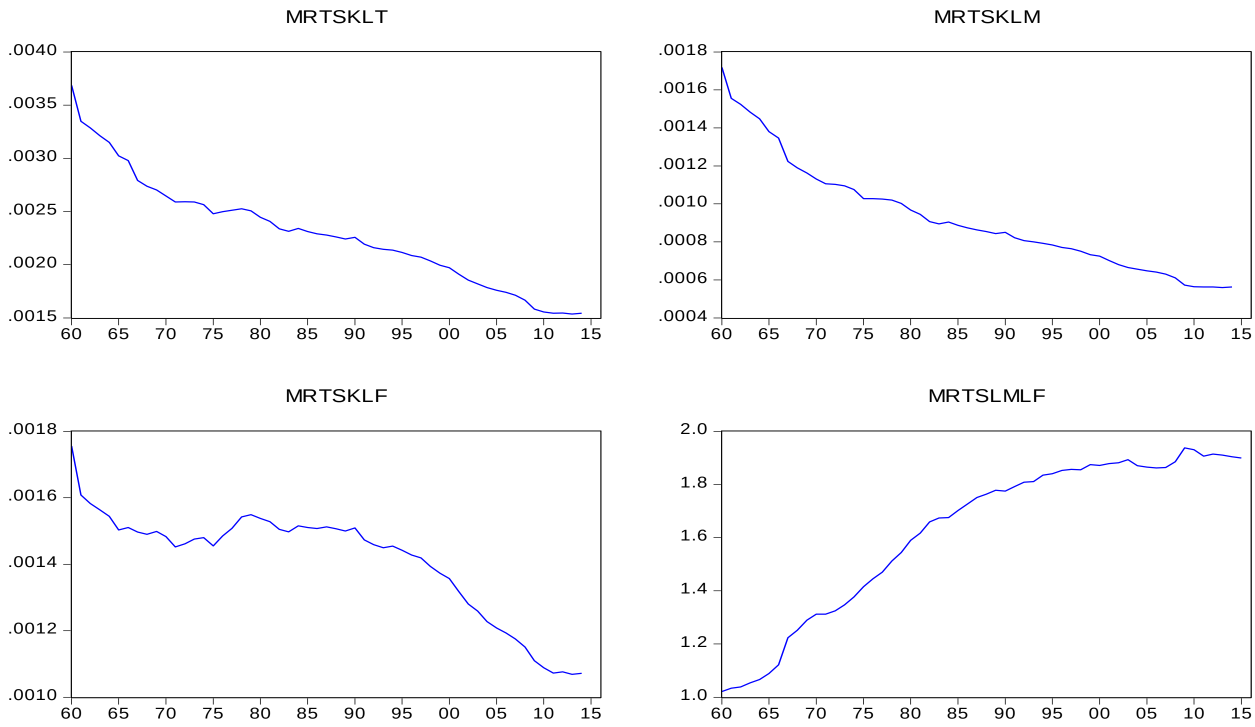

Figure 6. Marginal Rates of Technical Substitution of Capital for Aggregate/Gender Disaggregated Labor

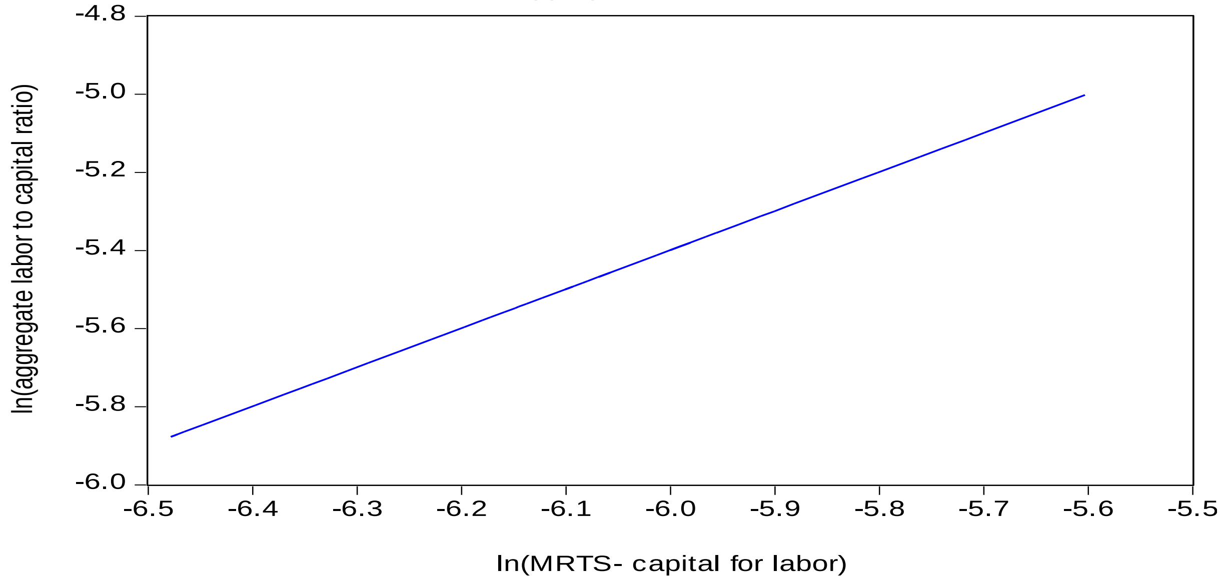

Figure 7a. Elasticity of Substitution of Aggregate Labor for Capital-Graphical Analysis

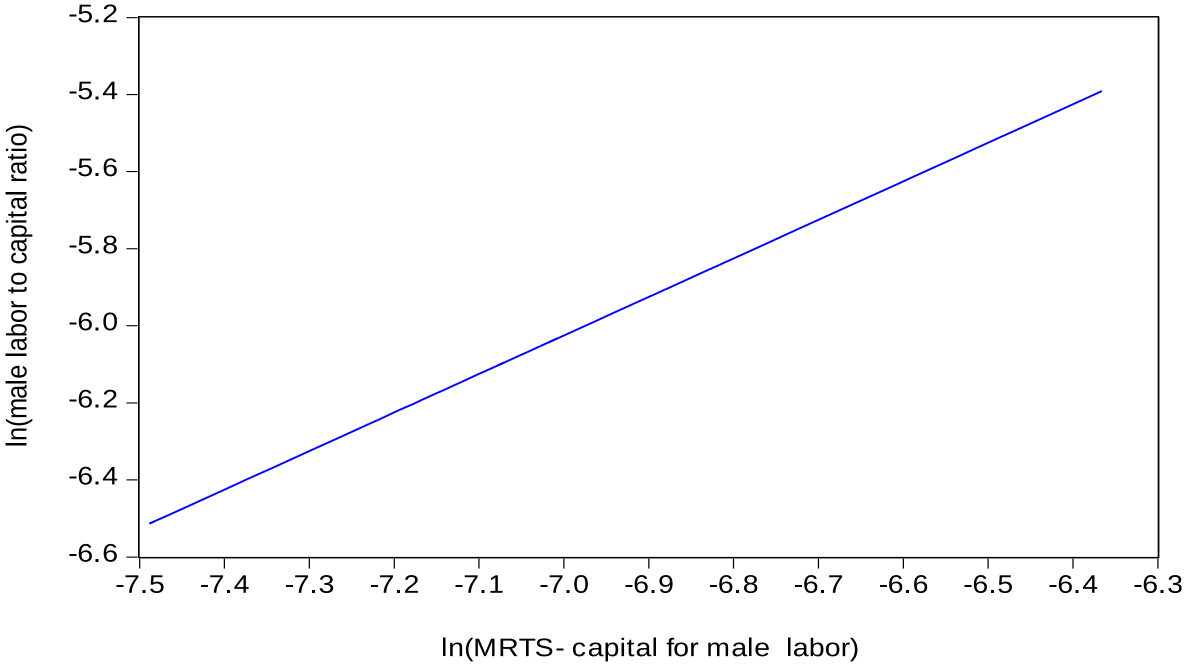

Figure 7b. Elasticity of Substitution of Male Labor for Capital-Graphical Analysis

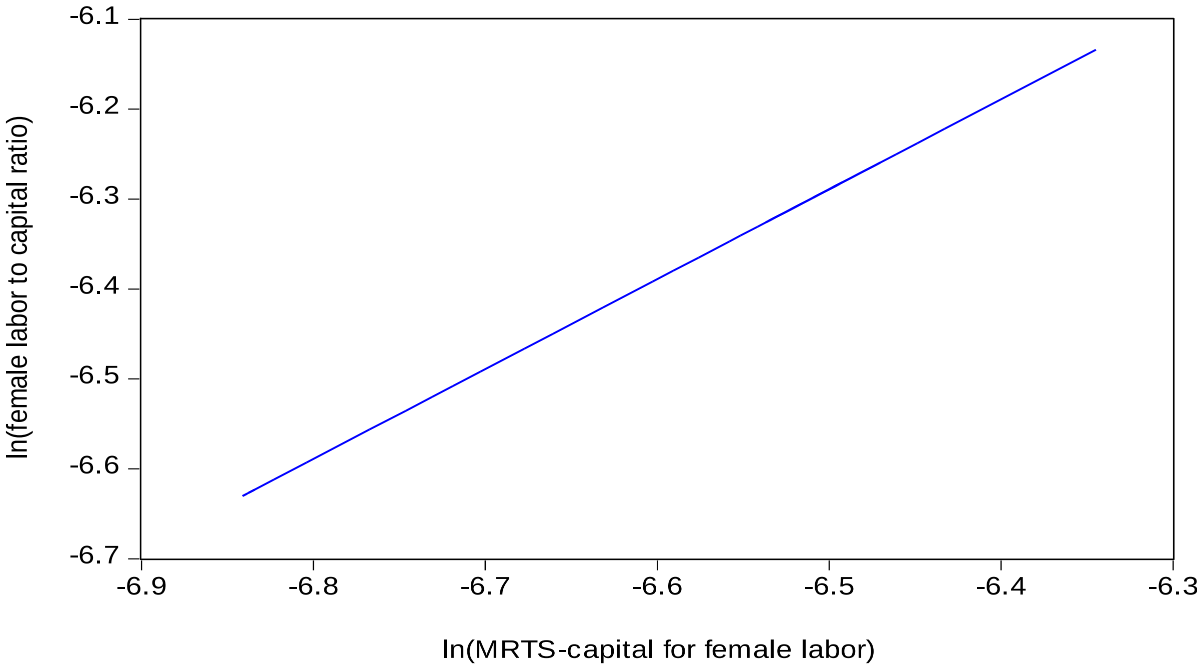

Figure 7c. Elasticity of Substitution of Female Labor for Capital-Graphical Analysis

Since women traditionally have less access to capital compared to men, this relatively lower substitutability (greater complementarity) of female labor for capital compared to male labor, may explain the lower levels of female productivity revealed by the corresponding partial proper elasticities of output. Also, this may explain, the high average elasticity of scale for female labor compared to men labor since the greater the number of women in the labor force, the greater their chances of accessing capital, which may be lacking otherwise due to gender discrimination that may occur in the capital markets. This implies that in order to increase output, it is necessary to ensure that both men and women can access the capital required for the different economic activities undertaken by each category, but due to the higher level of complementarity between female labor and capital compared to male labor, women would require more capital to work with, while men have a greater ability to substitute capital (lower complementarity). The ratio of average capital to the aggregate labor, was ranging between 148.6978 and 356.75 millions of USA dollars per unit of labor, with an average of 250.110 millions of USA dollars per unit of labor.

Further, the results (Table 10 and Figure 7d) also indicate that: the elasticity of substitution of female labor for male labor has remained constant over the period; and that the ease of substitutability between female labor and male labor is much smaller than the ease of substitutability between each of them and capital. This is probably due to gender gap in human capital development, with women lagging behind men in human capital investment-thus less qualified compared to men. This also signals the fact that the degree of complementarity between female labor and male labor is greater than the degree of complementarity between each of them and capital. This greater complementarity is probably explained by gender occupational segregation whereby women tend to have occupations that are supportive but lower paid to those undertaken by men, such as secretaries for male directors; and nurses for male medical doctors, among others. Ensuring gender equality in human capital and elimination of discrimination in the labor market can enhance the extent of substitutability between the two forms of labor in the professions dominated by each sex.

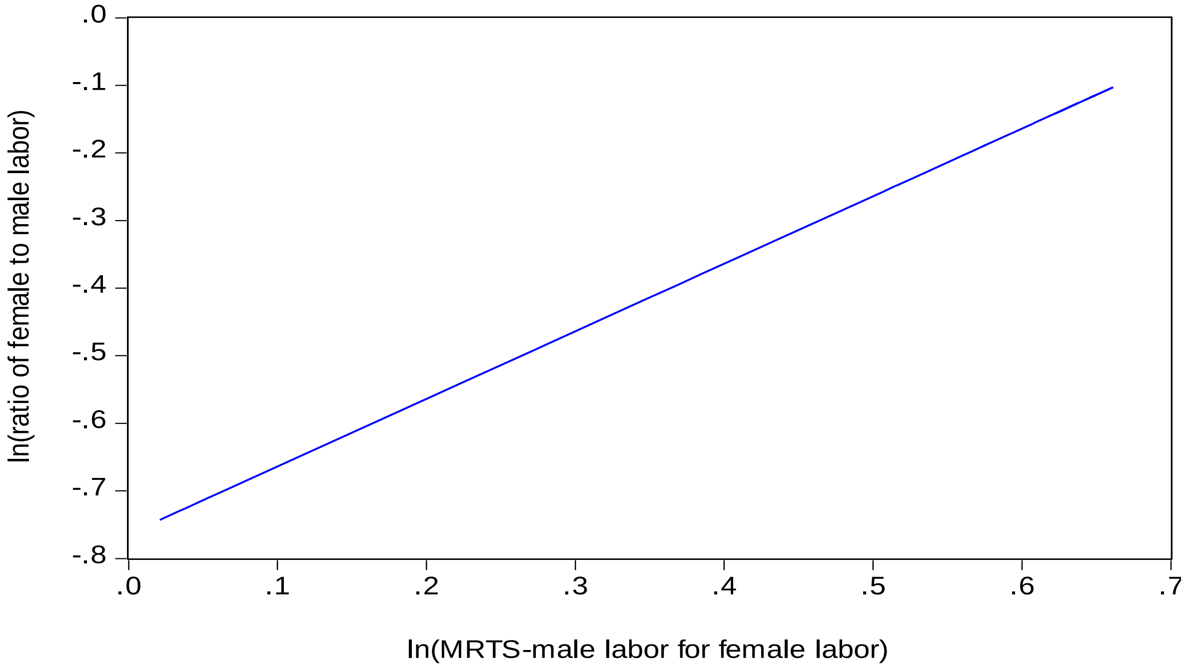

Figure 7d. Elasticity of Substitution of Female Labor for Male Labor-Graphical Analysis

Finally, these results show that labor and capital on one hand; and female labor and male labor on the other hand are neither perfect substitutes with linear production functions (constant and uniform MRTS or infinite elasticity of substitution), nor are they perfect complements-used in fixed proportions or Leontief production function (infinite or zero MRTS, or zero elasticity of factor substitutions). Also, the elasticities of substitution that are greater than one and less than infinity show that the appropriate production function is the CES function (with $0 \lhd \sigma_{ij} \prec \infty$). Also, this rules out the Cobb-Douglas production function since assumptions of the function are violated (lack of perfect substitutability since $\sigma_{ij} \neq 1$, constant returns to scale- elasticity of output greater than one-see results above etc.). In this case, the CES function could not be approximated by the translog multi-input function due to collinearity problems. The CES is the appropriate function regardless of the category of labor being investigated.

5.4 Testing for Cobb-Douglas production function

Due to existence of harmful collinear established for female labor, capital and total labor, the multi-input translog production functions, fails the test of no collinearity and thus the estimates of the translog production function would be invalid, thus making it redundant to perform further tests using the model, including the tests for choosing between this model and the Cobb-Douglas function.

To determine whether the CD production, both gender disaggregated and total labor model, are suitable functions, they were examined to ensure positivity of the function. This would be satisfied when all the marginal products estimated are positive. Since all the labor units and the output are positive, satisfying this condition is equivalent to requiring a positive elasticity of output for each of the inputs in the model.

Given the results in Table 3, for the gender disaggregated Cobb-Douglas function, the elasticity of output for capital (0.871) and male labor (0.755) are positive while that for female labor (-0.1017) is negative. The negative output elasticity of female labor would signal negative marginal products, thus violating the positivity requirement and thus the validity of the corresponding parameter estimates.

For the aggregate labor CD production function, the requirement for positivity is satisfied for both capital and aggregate labor. Based on this function, increasing returns to scale exist since

$\alpha_K + \alpha_{L_T} \succ 1$ ($\alpha_K + \alpha_{L_T} = 0.8905 + 0.3328 = 1.2233$). This implies that increasing the inputs by a certain proportion will disproportionately increase the output, thus the function is disproportionate and quickly growing. Since one of the underlying assumptions for a Cobb-Douglas production function is constant returns to scale (implying $\alpha_K + \alpha_{L_T} = 1$), which is violated in this case, it is concluded that the appropriate production function is not a CD production function.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

Based on the aggregate labor Cob-Douglas model, increasing returns to scale exists, which signifies violation of the constant returns to scale assumed for this function. Based on the labor productivity model, the study revealed that in the USA: aggregate labor productivity increases by 25.5% following a 1 % increase in female capita/female labor ratio but decreases by 1.98% following a 1% increase in the capital/male labor ratio (increase total capital outlays to increase female labor productivity, thus aggregate output.

Based on the single factor translog function, the study revealed that: increasing the capital/female labor ratio increases aggregate labor productivity but the reverse is true for capital/male labor ratio; male labor is more productive than female labor, which is more productive than capital with output elasticities of 2.827, 1.313 and 1.065, respectively; male labor has a higher proper output elasticity compared to female labor due to higher variability in productivity of the male labor, lower variations in the number of male labor units employed, a higher correlation between number of male labor employed and the productivity of male labor; and lower variability of the male labor force relative to output compared to their female counterparts; all the inputs considered have increasing returns to scale but GDP production is more dependent on labor compared to capital; and more on male labor compared to female labor; the augmented output elasticity is higher for female labor compared to male labor; a simultaneous increase in quantity allocated and productivity of each of the factors investigated leads to an increase in output; the growth in the marginal product (productivity) of female labor has been slower than that for the total labor force and that for male labor; labor, regardless of category considered, has been relatively more productive that capital, with greater increase in the relative productivity of male labor to capital compared to that for female labor (and total labor) to capital; female labor is about as half as productive as male labor (MRTS of 0.644 ($\pm$0.143), having declined from almost equally as productive (MRTS of 0.98 to 0.96) as male labor at the beginning of the period; the dynamic trajectory aggregate/male/female labor and capital are conventionally under-exponential; capital and labor, regardless of the category are neither perfect substitutes nor perfect complements, but the degree of substitutability between male labor and capital is greater than that between female labor and capital, female labor thus requires capital more as a complementary inputs; and The ease of substitution between labor and capital has remained constant for each labor category; the elasticity of substitution of female labor for male labor has remained constant over the period; and the ease of substitutability between female labor and male labor is much smaller than the ease of substitutability between each of them and capital.

Finally, it was noted that the increase in the female labor force relative to male labor force in the USA, has not been accompanied by an increase in the female labor productivity.

6.2 Recommendations

a) An increase in capital outlays in the USA, will have greater effect if directed to those sectors which are under-capitalized, particularly those dominated by female labor, since increasing the productivity of aggregate labor requires increasing the amount of capital available to the female labor.

b) The greater dependency of GDP growth on labor rather than capital in the USA, signals the need to increase productivity of the aggregate labor force (both male and female labor) in general, but with greater efforts tailored towards increasing the productivity of the current female labor force which is lower than that for their male counterparts to bring it at par with their male counterparts.

c) Efforts to increase the output elasticity (productivity) of the female labor force in the USA, should be aimed at increasing the variability of productivity of female labor, for example by reducing the concentration in female dominated occupations and sector through human capita development; reducing the variation of the number of female labor employed, for example by ensuring that appropriate safety nets are put in place to ensure that women do not leave the labor force to cater for family responsibility or designing work schedules that can allow them to cater for both unpaid care and paid work; increasing the correlation between number female employees and their productivity, for example by providing the capital base required in many of the activities undertaken by women; and reducing the variability of female labor force.

This requires clear understanding of the constraints faced by women and the possible solutions (See Bradshaw, Castellino and Diop [2013] for suggestions on priority areas to address different constraints.

d) A simultaneous increase in quantity allocated and productivity of each of the factors investigated leads to an increase in output, measures to increase productivity should be accompanied with measures to increase the quantity allocated. For gender disaggregated labor, this implies that measures to increase female/male labor productivity, should be accompanied with measures to increase gender disaggregated participation rates, with more emphasis on female labor participation rates which still lag behind those for men. Thus, efforts towards attaining gender parity in numbers in the labor force should be accompanied with efforts to achieve gender parity in productivity.

e) Slower growth in the productivity of female labor underscores the need for measures that allow the female labor productivity to grow in tandem with the male labor productivity by having measures that reduce their burden, particularly of the unpaid care work which is, in most cases, is not accounted for in the national output.

f) To eliminate the gender wage gap, interventions that can increase the productivity of female labor relative to male labor should be implemented.

g) Greater complementarity between female labor and capital compared to male labor implies that to increase the productivity of female labor, it is necessary to increase the capital in those sectors where women still dominate the production process in the short-run, while in the long-run efforts are made to increase the number of women engaged in the sectors where it is much easier to substitute labor for capital, mainly the professional jobs which tend to be dominated by men and are higher rewarding.

h) Measures aimed at increasing the ease of substitutability between male labor and female labor, such as those aimed at closing the gender gap in human capital development and reducing discrimination, should be implemented.

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